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RESEARCH PAPER

Upper Bounds of Stock Portfolio Investment Risk Using Value at Risk (Case Study: Indonesian Blue-Chip Stocks in 2022)

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Abstract
In recent years, stocks become the most preferred asset by Indonesian investors. Besides offering large profits, stock investment also has a risk factor that can occur at any time. One way to minimize risk is to form a stock portfolio. This paper aims to measure the upper bounds of the portfolio loss risk formed by several single assets that are mutually dependent. The upper bound value is chosen because the exact value of portfolio loss risk is difficult to obtained by Convolution or Panjer Recursion methods. The main analysis of this research is formed the upper bounds of stock portfolio investment risk using VaR with Cornish Fisher Expansion aproach by utilized comonotonicity and convex order properties. The portofolio contains of 3 single asset (ARTO.JK, ITMG.JK, and MIKA.JK) which collected from IDX Indonesia from 10/25/21 to 10/21/22. The novelty of this research is combined comonotonicity and convex order properties with VaR-CFE to get upper bounds of portolio risk predicition. The result show that at 95% significance level and 1-day holding period, the upper bounds of VaR-CFE prediction for the portfolio is -0.1394. The social impact of this research can be a benchmark to get accurate risk prediction of their portfolio asset.
Keywords: Comonotonicity, Convex Order, Portfolio, Risk, Value at Risk-Cornis Fisher Expansion.
JEL Classification: G11, G32.

1. Introduction
Investment, as economic activities of Indonesian having various backgrounds (Fernandez et al., 2020), has been chosen to meet their needs and prepare for a

better life in the future (Maruddani and Trimono, 2021). According to (Ridha and Budi, 2020), apart from being able to improve people's living standards, investment activities can also encourage national economic growth by increasing the GDP value. Murti and Sahara (2019) stated that countries with a favorable investment climate tend to have better economic growth than countries with a stagnant investment climate.

Financial assets traded on the IDX Indonesia including stocks, options, futures, mutual funds, and bonds (Tarno et al., 2020). According to the Indonesian Securities Custodian (KSEI), during 2017-2022, there was a significant increase in the number of official investors registered with IDX Indonesia (KSEI Indonesia., 2022a). In 2017, the number of registered investors was 1,000,289 (KSEI Indonesia, 2017), then in November 2022, it rise to 10,000,628 (KSEI, Indonesia, 2022b), or increased by 999.77%. This trend shows the increasing public interest to invest in financial assets. Stocks are the assets most in demand by investors. As of November 2022, 4,323,643 of 10,000,628 investors in IDX Indonesia were stock investors (KSEI Indonesia., 2017).

In order to improve the performance of each company and provide a reference for potential investors, IDX Indonesia releases a list of stocks of those categorized as blue-chip. According to (Maruddani and Trimono, 2021), blue-chip stocks are the most recommended stocks for investors because they provide stable profits. Jagati (2019) suggested that beginner and middle-class investors with investment funds between IDR 10 million and 100 million choose blue chip stocks, as they have a stable performance and high selling power. Thus, if investors want to resell their own stocks, finding potential buyers will not be difficult.

According to Murata and Hamori (2021), stock investment also has a risk factor for losses that can occur at any time apart from offering large and relatively fast profits. In stock trading activities, stock prices often experience fluctuations caused by various factors. Therefore, investors must choose the right investment strategy to maximize profit with the possible minimum risk (Avramov et al., 2013). One way to minimize risk in stock investment is to diversify. The diversification process involves forming an efficient stock portfolio with minimum risk. According to Radović, Radukić and Njegomir (2018), an efficient portfolio can produce optimal profit levels with the lowest loss risk.

However, the formation of a portfolio is only to minimize risk, not to eliminate it (Zanfelicce and Rabechini Jr, 2021). Consequently, investors need to know the estimated limit value of losses that may occur to maintain investment stability and prevent investments from bankruptcy. Meanwhile, according to

(Trimono et al., 2019), predicting the loss risk can be done using the VaR. VaR predicts an asset's maximum loss value under normal market conditions at a certain confidence level and period. In measuring portfolio risk using VaR, the distribution function of the portfolio is needed (Tarno et al., 2022a). However, it is difficult to determine the joint distribution function of two independent random variables (Jakobsons et al., 2016). Therefore, an alternative to calculating the estimated risk value of a stock portfolio in which its single assets are not mutually exclusive is calculating the upper bound of the portfolio risk value without using the joint distribution function (Xing and Li, 2018). Furthermore, the value of the upper bound for the loss risk obtained can be used as a reference by investors to prepare actions and strategies that must be taken if one day the risk occurs (ARAI, 2017).

The aim of this study was to establish an upper bound on the loss risk in a blue-chip stock portfolio at IDX Indonesia with the single stocks making up the portfolio that is not mutually dependent (mutually dependent). The upper bound model was developed by utilizing the komonotonic and convex order characteristics of the single stock returns that make up the portfolio. There were two novelties offered in this study. The first was using the VaR with Cornish Fisher Expansion (CFE) approach to calculate the value of the upper bound of losses on a stock portfolio. In previous research on determining VaR as the upper bounds of portfolio risk (i.e., Bernard et al., 2017; Ansari and Rüschendorf, 2020), there was no explanation of the construction of the VaR model using the CFE approach. The next novelty was the involvement of three mutually dependent single assets with a negative correlation as portfolio constituent variables. Some previous research on this topic used two variables for the numerical simulation; these studies include Jakobson et al. (2015) and Maggioni and Pflug (2019).

2. Literature review

In determining the upper bound of loss risk for the IDX Indonesia blue chip stock investment portfolio, this study referred to several relevant similar studies to obtain optimal research results. Several previous studies related to risk analysis of stock investment portfolios include; Hadiyoso, Firdaus and Sasongko (2016) used the single index model to develop an optimal portfolio of 43 Islamic stocks at IDX. In a numerical simulation, using historical price data from /05/12/11 to 07/04/14, the result showed that the most significant proportion of the portfolio is MKPI.JK with value of 11.32%, while the smallest one is KOIN.JK (0.05%). The stock that have a large proportion of the formation of the portfolio (more than 5%) primarily stocks from the trade, property, and basic chemical industry sectors. However, an analysis of the loss risk was not available, considering that the risk was also an essential indicator in the portfolio. Thus, it needed to be further studied.

Sumaji, Hsu and Salim (2017) analyzed market risk values using VaR on nine manufacturing company stocks listed on IDX LQ-45. The result showed that the stock portfolio was formed based on the Markowitz method without regard to the independence between stocks. The analysis showed that the VaR model with the variance-covariance approach is the best model for measuring the maximum potential loss of a manufacturing stock portfolio. Salsabila and Hasnawati (2018) investigated the value of portfolio risk in the Indonesian Companies Listed on the LQ45 Index for the 2013–2016 period. They constructed the portfolio based on the Markowitz method without examining the dependencies of every single asset making up the portfolio. Then, the portfolio formed was used as a reference for predicting the value of the loss risk.

Next, Pasaribu (2019) explored the amount of loss risk in a portfolio composed of liquid stocks on the Indonesian stock market. However, this study did not specifically explain the procedure for compiling a stock portfolio; they used a portfolio determined in another study and then analyzed the value of the loss risk using VaR. Juniar et al. (2020) formation of an optimal portfolio with the Markowitz model and analyzed the loss risk based on VaR for 10 Sharia stocks in the Jakarta Islamic Index. In this study, there was no dependency test on each stock, so there was no conclusion about whether single asset were mutually dependent or mutually independent. Loss risk analysis of Portfolio formation was carried out using VaR-MCS. The results showed that for a single asset, at a 99% confidence level and a one-day holding period, the most significant loss occurred in ADRO.JK, which was -7.24%. VaR MCS predicted a loss of -4.32% in the stock portfolio with the same confidence level and holding period. The result suggested that the portfolio can minimize the value of the loss risk.

KSEI Indonesia. (2022b) reported that banking stocks are among the most in demand by investors. Referring to this fact, Sholikhah, Sudarto and Shaferi, (2020) analyzed banking stocks by forming a portfolio to minimize the loss risk. Based on the stock price of 10 banks in Indonesia in March - July 2020, the MVEP method resulted that stocks with the highest proportion were BBNI.JK (20.23%) and stocks with the lowest proportion were BBHI.JK (1.32%). This study did not include the process of preparing a portfolio, so it was likely that the results obtained needed to be more accurate. Research related to Indonesian banking stocks was also conducted by Irsan, Priscilla and Siswanto (2022). They particularly examined the risk value using the VAR model with a historical simulation and variance-covariance approach. The portfolio weight was determined using the MVEP Method. The drawback of this study was not to test the dependence on three banking stock prices that were the research sample. The results showed that in the 95% confidence level and holding period of one day, the results of the prediction of losses by holding variance-covariance and historical simulation were -0.02790 and 0. 01978.

According to Embrechts et al. (2013), to measure the loss risk in a stock portfolio, a distribution function of portfolio return is needed, where the distribution function of portfolio return is a joint distribution function of single asset return. If the return price of a single asset is mutually independent, then determining the distribution function of portfolio returns is easy to determine (Bernard et al., 2018). Nevertheless, the distribution function of portfolio returns will require more work to determine if single-asset returns are mutually dependent (Bignozzi et al., 2015). To solve this problem, Pucetti (2013) proposed an alternative to calculating the upper bound of portfolio loss risk without using the portfolio distribution function. Chen et al. (2022) guaranteed that the upper bound obtained will always be more significant or at least the same as the actual loss risk.

Based on previous research, this research predicted the loss risk in a stock portfolio composed of 3 stock assets that were not mutually exclusive. The risk measure used was Value-at-Risk (VaR) with the Cornish Fisher Expansion (CFE) approach. The stocks used in this study were three blue chip stocks at IDX Indonesia, which were not mutually exclusive with negative correlation values. This proposition was based on the findings of Achudume and Ugbebor (2021). They suggested that a portfolio will be optimal if every single asset composed of it is negatively correlated so that if one stock suffers a loss, other stocks can still cover the loss.

3. Data and Methods

3.1 Data Description

In this study, an analysis of the upper bound of VaR on the stock portfolio used the stock data of Blue-chip companies at IDX Indonesia in 2022, which were Jago Bank (ARTO.JK), Indo Tambangraya Megah (IMTG.JK), and Mitra Keluarga Karyasehat (MIKA.JK). They were selected following the concept of forming a portfolio put forward by (Tarno et al., 2020) and (Tarno et al., 2022b) that the main objective of forming a portfolio was to optimize the return value by reducing the loss risk to as small as possible. In achieving the goal, the portfolio's single asset returns had to be negatively correlated. So, one day, one of the assets suffered a loss. In that case, there was still a significant possibility that the other asset would record a profit.

3.2 Convex-order

Convex order is one of the methods used to compare two random variables with identical and finite expected values. The following is the convex-order definition for a single random variable, according to (Lu et al., 2018).

Definition 1. [Convex-order] Let X_1 and X_2 be two random variables, and g is a convex function such that the values $E[g(X_1)]$ and $E[g(X_2)]$ are defined. X_1 is said to be smaller than X_2 in convex-order ($X_1 \leq_{co} X_2$) if $E[g(X_1)] \leq E[g(X_2)]$.

According to Florea et al. (2015), Convex-orders are closely related to stop-loss orders; this can be seen from a bi-implicated relationship between the two. The definition of stop-loss-order for two single random variables, according to Bouhadjar et al. (2016), is as follows:

Definition 2. Suppose there are two random variables, X_1 and X_2. X_1 is said to be smaller than X_2 in the stop-loss-order ($X_1 \leq_{\text{sl}} X_2$) if for every constant K holds:

$$E\left[\left(X_1 - K\right)_+ \leq E\left(X_2 - K\right)_+\right] \text{ and } E[X_1] = E[X_2] \quad (1)$$

Referring to Jain and Gupta (2018), apart from the bi-implication relationship, several relationships between convex orders and stop-loss orders are as follows: (i) if $X_1 \leq_{co} X_2$, then E[X_1] = E[X_2]; (ii) If $X_1 \leq_{co} X_2$ then $X_1 \leq_{sl} X_2$; (iii) $X_1 \leq_{sl} X_2$ then $X_1 \leq_{co} X_2$.

3.3 Commonotonic Random Variables

Suppose we have a set X which contains some N random variables, namely, X_1, X_2, …, X_N. A is a commonotonic set if one of the following equivalent statements is fulfilled (Gao and Zhao, 2017): (1) X has commonotonic-support. A commonotonic set A $\subset \mathbb{R}^N$ is called commonotonic-support of X if P[X $\in$ A] = 1. (2) For every x = $(x_1, x_2, \ldots, x_n)$, then F_X(x) = min $\left\{F_{X1}(x_1), F_{X2}(x_2), \ldots, F_{Xn}(x_n)\right\}$ (3) For random variable U ~ Uniform (0,1), valid X similar in distribution to $\left(F_{X1}^{-1}(U), F_{X2}^{-1}(U), \ldots, F_{XN}^{-1}(U)\right)$; (4) There is a random variable Z and an undecreasing function φ_i such that X is similar in distribution to $\left(\varphi_1(Z), \varphi_2(Z), \ldots, \varphi_N(Z)\right)$.

According to Kumar and Srinivasan (2014), one of the characteristics of the random variable is having a distribution function that plays a vital role in predicting risk values. Suppose X is a single random variable with the $F_X(\cdot)$ distribution function. Based on Cheung and Vanduffel (2013), for each random variable X, there is X^C which is the Commonotonic Random Variable of X, and there is φ which is a non-degenerate function, and random variable Z so that X^C is similar in the distribution of $F_X^{-1}(U)$. As U is distributed Uniformly (0,1), in the end, the values of $F_X^{-1}(U)$ are the values of the random variable X itself. As a result, X is similar in distribution to X, and the distribution function of X^C is the same as the distribution function X, namely $F_X(\cdot)$ (Ortega-Jiménez et al., 2021).

The Upper Bound in Convex-Order for Stock Portfolio

According to Jakobsons and Vanduffel (2015), a convex-order bound applies for each random variable X, which is a single asset return, and S_N, which is a portfolio return of N stock assets. Furthermore, the upper bound in the convex order will be a reference for measuring the bound of the loss risk for the S_N. portfolio. For X_1, X_2, …, X_N, which are returns from some N stocks, S_N is obtained through the following equation (R. Zhou and Palomar, 2021):

$$S_N = X_1 + X_2 + \ldots + X_N \tag{2}$$

Bernard et al. (2017) state that the upper bounds in the convex-order sequence for X and S_N are X^C and S_N^C. X^C is the commonotonic random variable of X, while S_N^C is the commonotonic random variable of S_N. So, mathematically, it can be written as:

$$S_N \leq_{co} S_N^C \tag{3}$$

Inequality (3) implies S_N^C also being the upper bound of the stop-loss order for S_N, so it can be written as follows:

$$S_N \leq_{sl} S_N^C \tag{4}$$

According to Haugh et al. (2015), since every X_i is guaranteed to have X_i^C as a boundary in the convex-order, then S_N^C can be obtained by adding up each X_i^C value, namely:

$$S_N^C = X_1^C + X_2^C + \ldots + X_N^C \tag{5}$$

Then, the inverse distribution function for from S_N^C at point α ∈ (0,1), i.e. $F_{S_N^C}^{-1}(\alpha)$ is given by the following theorem:

Theorem 1. (Sun X. et al., 2018) inverse distribution function for from S_N^C at point α ∈ (0,1), defined as:

$$F_{S_N^C}^{-1}(\alpha) = \sum_{i=1}^{n} F_{Xi}^{-1}(\alpha) \tag{6}$$

The Upper Bound of VaR on the Stock Portfolio

In stock portfolios, a risk measure can be used to predict the loss risk that might occur in the future (Zhou et al., 2018). In this study, the risk measure used is VaR-CFE. Theoretically, VaR is the maximum risk value that can still be tolerated at a confidence level of 1-α, α ∈ (0,1) with a specific holding period. Suppose S_3 is a random variable representing the portfolio return of 3 stocks, VaR for S_3 at a confidence level of (1 – α) is defined as (Alshamali et al., 2021):

$$VaR_{1-\alpha}(S_3) = F_{S_3}^{-1}(\alpha) \tag{7}$$

that is, $F_{S_3}^{-1}(\cdot)$ is the inverse of the distribution function of $F_{S_3}(\cdot)$.

According to Feng, Wächter and Staum (2015), to predict the value of $\mathrm{VaR}_{1-\alpha}(S_3)$, a distribution function $F_{S_3}(\cdot)$ and an inverse distribution function $F_{S_3}^{-1}(\cdot)$ are needed. If X_1, X_2 and X_3 are single asset returns that makeup mutually independent portfolio returns, then $F_{S_3}(\cdot)$ is relatively easy to determine using the Convolution or Panjer Recursion method (Cooper et al., 2021). However, if X_1, X_2 and X_3 are mutually dependent, $F_{S_3}(\cdot)$ and $F_{S_3}^{-1}(\cdot)$ become difficult to determine. As a result, the VaR model cannot be constructed, and we cannot estimate the loss risk. To overcome this problem, (Hanbali et al., 2022) introduced another alternative to predict the loss risk, namely by determining the upper bound value for $VaR_{1-\alpha}(S_3)$. The upper bound of $VaR_{1-\alpha}(S_3)$ is given by the following theorem:

Theorem 2. (Dhaene et al., 2014) for every random variable *Y* and *Z*, apply:

$$Y \le_{sl} Z \Leftrightarrow VaR_{1-\alpha}(Y) \le VaR_{1-\alpha}(Z) \tag{8}$$

which applies to every α ∈ (0,1).

In previous section, it has been explained that for each S_3, one can construct S_3^C, a commonotonic random variable of S_3, as well as an upper bound for S_3 in convex orders and stop-loss orders. Therefore, based on Theorem 2, it is obtained

$$VaR_{1-\alpha}(S_3) \leq VaR_{1-\alpha}(S_3^c) \quad (9)$$

So, the upper bound of portfolio loss risk is the VaR value of S_3^C. Then, the $\text{VaR}_{1-\alpha}(S_3^C)$ construct is determined based on the following theorem:

Teorema 3. (Cheung et al., 2017) Consider $X_1, X_2, \ldots X_N$ as commonotonic random variable, and suppose:

$$S_N^c = X_1^c + X_2^c + \ldots + X_N^c \quad (10)$$

Then for every α ∈ (0,1), the additive properties of VaR apply, namely:

$$VaR_{1-\alpha}(S_N^c) = VaR_{1-\alpha}(X_1) + VaR_{1-\alpha}(X_2) + \ldots + VaR_{1-\alpha}(X_N) \quad (11)$$

For a portfolio composed of 3 single assets, the predicted value of losses using the VaR method at a confidence level of 1-α will not be greater than the sum of $VaR_{1-\alpha}(X_1)$, $VaR_{1-\alpha}(X_2)$, and $VaR_{1-\alpha}(X_3)$. Alternatively, if written in the equation is as follows:

$$VaR_{1-\alpha}(S_3) \leq VaR_{1-\alpha}(X_1) + VaR_{1-\alpha}(X_2) + VaR_{1-\alpha}(X_3) \quad (12)$$

Cornish Fisher Expansion (CFE) Approach on VaR Method

Amédée-Manesme et al. (2019) argue that if stock returns do not precisely follow the normal distribution (skewness = 0 and kurtosis = 3), then the results of calculating VaR with the HS, VC, and MCS approaches will be inefficient. To overcome this problem, the CFE method can be used as an alternative to calculating VaR. CFE tends to provide a larger estimate of the VaR value. However, the VaR estimate will be smaller, especially if the data return has a positive slope. Specifically, ECF considers skewness and kurtosis values in VaR calculations (Christoph et al., 2020).

By using the CFE approach, the α-th used for VaR measurements is expanded by the following formula (Rastegar and Amzajerdi, 2019)

$$\text{CFE}= q_\alpha + \frac{\left((q_\alpha)^2 - 1\right)S(X)}{6} + \frac{\left((q_\alpha)^3 - 3q_\alpha\right)\psi(X)}{24} - \frac{\left(2(q_\alpha)^3 - 5q_\alpha\right)S^2(X)}{36} \quad (13)$$

If the kurtosis value is less than 3, then the CFE formula is as follows:

$$\text{CFE}= q_\alpha + \frac{\left((q_\alpha)^2 - 1\right)S(X)}{6} + \frac{\left((q_\alpha)^3 - 3q_\alpha\right)K(X)}{24} - \frac{\left(2(q_\alpha)^3 - 5q_\alpha\right)S^2(X)}{36} \quad (14)$$

where, CFE is the Cornish-Fisher expansion value; q_α is the α-the quantile of Standard Normal distribution; *S(X)* is the skewness of stock returns; *K(X)* is the kurtosis of stock returns; $\psi(X)$ is the difference in excess kurtosis values. If the initial investment value is V_0 and the holding period is *t*, then the VaR formula with the CFE approach at (1 – α) confidence level can be calculated as (Westgaard et al., 2020):

$$VaR_{\alpha}^{CFE}(X) = -V_0 \times (\mu - CFE\,\sigma) \times \sqrt{T} \tag{15}$$

with, μ is stock return mean, and σ is stock volatility.

Risk Prediction Accuracy with Backtesting Test

The concept of accuracy testing in the backtesting method calculates the violation ratio (Zhang and Nadarajah, 2018). In the period K_{E+1} to period K_U (length of the test window), the violation is symbolized by η_k, which is worth 1 if there is a violation and 0 if there is no violation in period k. Rt is the return in period *k*, and Adj-Esk is the value of Adj-ES in period *k*. The number of violations is symbolized by v_j with j = {0, 1}, where v_1 is the number of η_k which is worth 1 (the number of days the violation occurred), and v_0 is the number that is worth 0 (the number of days without a violation).

$$v_1 = \sum_{k=K_{E+1}}^{K} \eta_k \text{ and } v_0 = K_U - v_1 \tag{16}$$

The violation ratio (*VR*) is calculated by comparing v_1 with the expected number of violations. Where m_0 is the probability of the estimated violation, then

$$VR = \frac{v_1}{m_0 \times K_U} \tag{17}$$

If VR = 1 or VR < 1 then the risk prediction is accurate, meanwhile if VR > 1 then the risk prediction isn't accurate.

4. Result and Discussion

The historical stock price data that became the sample was from 10/25/21 to 10/21/22. So, with each variable, there will be 246 data. The following is a descriptive statistical value of historical stock price:

Table 1. Descriptive Statistics of Daily Historical Stock Price

	Mean	St. dev	Min	Max	Skewness	Kurtosis
ARTO.JK	12208.90	3846.02	4680	19000	-0.05798	-1.38039
ITMG.JK	29913.16	7947.07	19100	44175	0.31826	-1.22953
MIKA.JK	2481.88	234.61	2090	3050	0.08202	-1.22934

Soruce: Research finding.

Referring to the standard deviation value, the stock with the highest deviation from the average value was ITMG.JK. Meanwhile, the stock with the most homogeneous value and the lowest deviation was MIKA.JK. Similarly, based on the skewness value, the ITMG.JK and MIKA.JK stock price data tended to converge at a value smaller than the average because the two stocks had a positive skewness value. For ARTO.JK, the negative skewness value meant that the stock price data converged during the observation period at a value greater than the average. The kurtosis of all assets was less than 0. This state suggested that the distribution curve was flatter than a normal curve with the same mean and standard deviation. During the observation period, the three stocks experienced various price fluctuations. To find out the fluctuation pattern, we present a time series plot in the following figure:

Figure 1. Time Series Plots of Daily Historical Stock Price at Observation Period
Soruce: Research finding.

During the observation period, the stocks price experiencing the most significant price decline were Arto.JK. At the beginning of the period (10/25/21), the stock price was at the IDR 14,925 level; this value dropped to IDR 5.425 in the final recording period. Referring to Detik Finance, the main factor causing this decline was an increase in the benchmark interest rate issued by Bank Indonesia (Finance Detik, 2022). Unlike Arto.JK, two other stocks price experienced a significant increase. For MIKA.JK, the highest increase occurred around March - May 2022. As for ITMG.JK, the increase occurred more slowly from the beginning to the end of the recording period.

We need historical return values to construct the VaR model on upper-bound portfolio loss risk prediction. The return value is obtained in this study using the Log return method. Suppose there are P_t and P_{t-1}, which are stock prices in periods *t* and *t*-1, then the return value for period *t* can be obtained through the following equation:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{16}$$

Descriptive statistical values for each single asset return are presented in the following table:

Table 2. Descriptive Statistics of Daily Historical Return

	Mean	St. dev	Min	Max	Skewness	Kurtosis
ARTO.JK	-0.00415	0.03888	-0.07201	0.18055	0.75772	2.11977
ITMG.JK	0.00231	0.02798	-0.07235	0.11790	0.41569	1.49769
MIKA.JK	0.00104	0.02289	-0.07134	0.07232	-0.00222	1.51736

Soruce: Research finding.

During the observation period, the stock that provided the most considerable average daily profit was ITMG.JK, which was engaged in coal mining. Coal was one of the largest mining commodities exported by Indonesia, and its price tended to increase during the observation period. The increased price was due to plans to lift the ban on coal imported from Australia, increase the capacity of coal rail transport, and improve logistics conditions because the weather was back to normal (Kompas, 2021). Furthermore, the stock with the lowest average return was ARTO.JK (-0.415%). This condition showed the value of the losses recorded by ARTO.JK during the observation period was more significant than the value of the profits received. The fall in digital bank stocks, such as ARTO.JK., was caused by expensive valuations compared to other big banks. Then, the average profitability of digital bank issuers was still relatively small. The highest profit values were recorded during the observation period by ARTO.JK, ITMG.JK, and MIKA.JK was 18.055%, 11.790%, and 7.23%, respectively. Referring to the Skewness value, ARTO.JK and ITMG.JK returns tended to center on a value that was smaller than the average; this was because the skewness of both was positive.

In determining the upper bound of the portfolio loss risk, the first procedure was to determine the portfolio model used. Because our goal was to determine the upper bound of the portfolio loss risk in general, the portfolio model we choose was the general model, namely:

$$S_{3,t} = R_{1,t} + R_{2,t} + R_{3,t} \tag{17}$$

with, $S_{3,t}$ is the portfolio return in period *t*. $R_{1,t}$, $R_{2,t}$, and $R_{3,t}$ respectively were the stock returns of ARTO.JK, ITMG.JK, and MIKA.JK in period *t*. Furthermore, before carrying out the correlation test, we performed a normality test for every asset return. The purpose of the normality test was related to selecting the best correlation test method to be used. If every asset return was normally distributed, the best correlation test method was Pearson correlation, otherwise, we could use the Spearman method (Aghnitama et al., 2021). By using α = 5%, the normality test results using the Kolmogorov-Smirnov test are as follows:

Table 3. Normality Test of Daily Historical Return

	KS value	Sig	Decision
ARTO.JK	0.978	0.294	The data are normally distributed
ITMG.JK	1.207	0.109	The data are normally distributed
MIKA.JK	1.004	0.265	The data are normally distributed

Soruce: Research finding.

Table 3 shows that the value of each single asset return is sig > α; therefore, we concluded that daily historical returns were normal. Since the data were normally distributed, the correlation test method chosen was Pearson's correlation. The Pearson correlation method calculates the correlation value based on the product moment of each data value on the corresponding variable. The statistical test values for the Pearson correlation test are given in the following table:

Table 4. Pearson Correlation Test Statistical Test Scores

	ARTO.JK	ITMG.JK	MIKA.JK
ARTO.JK	$r = 1$	r = -0.133 sig = 0.037	r = -0.126 sig = -0.049
ITMG.JK	r = -0.133 sig =0.037	$r = 1$	r = -0.163 sig = 0.011
MIKA.JK	r = -0.126 sig = 0.049	r = -0.163 sig = 0.011	$r = 1$

Soruce: Research finding.

Table 4 shows a significant negative correlation between the three stock returns; therefore, the stock portfolio formed was ideal based on the rules for compiling a portfolio according to (Hersugondo et al., 2022). Before determining the upper bound for S_3, we present the value of descriptive statistics for S_3 to obtain characteristics and descriptive information before we determined the upper bound of losses using VaR.

Table 5. Descriptive Statistics of Portfolio Return

	Mean	St. dev	Min	Max	Skewness	Kurtosis
Portfolio Return	0.08279	0.04578	-0.11208	0.15504	0.03070	-0.08684

Soruce: Research finding.

Table 5 exhibits that the formation of a portfolio can produce an average positive return and suggests that during the observation period, the profits obtained by investors were worth more than the losses received. The most significant loss value received on portfolio investment was -0.11208. However, this loss's value was smaller than the maximum profit value of 15,504%. In other words, the maximum profit earned could still cover the losses incurred. The distribution of portfolio return data tended to be centered on values below the average; this condition referred to skewness with a positive value. Furthermore, the most centered data distribution was in the value interval 0.03421 – 0.07882, with a frequency of 50 data. Furthermore, the time series plot of stock return is given in the following figure:

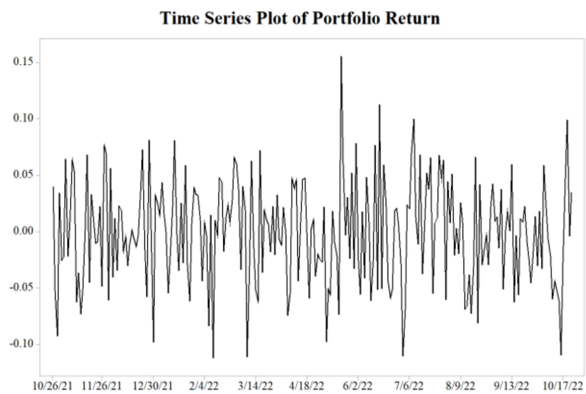

Figure 3. Time Series Plots of Daily Portfolio Return at Observation Period
Soruce: Research finding.

Figure 3 shows that the portfolio return is stationary during the observation period. It means there are no extreme jumps in value or outlier data. The highest profit value was recorded on 05/19/22, while the biggest loss occurred on 02/12/22. The previous chapter explained that the upper bound for portfolio losses using the VaR risk measure was the sum of the VaR values of each single asset return. The advantage of this method was that we did not need to construct a portfolio return distribution function, so the processing steps became efficient. Theoretically, the results obtained are guaranteed to be accurate.

The Upper Bound of Portfolio Loss Risk Using VaR with Cornish-Fisher Expansion (CFE)

For example, P_3 is a portfolio composed of ARTO.JK, ITMG.JK, and MIKA.JK stocks and S_3 is a portfolio return. In the *t*-period, the upper bound of the VaR of $P_{3,t}$ at each confidence level (1 – α) and the holding period *w* is obtained using the following formula:

$$VaR^{CFE}_{(1-\alpha),w}(S_3) = VaR^{CFE}_{(1-\alpha),w}(R_1) + VaR^{CFE}_{(1-\alpha),w}(R_2) + VaR^{CFE}_{(1-\alpha),w}(R_3) \quad (18)$$

with $VaR^{CFE}_{(1-\alpha),w}(R_1)$, $VaR^{CFE}_{(1-\alpha),w}(R_2)$, and $VaR^{CFE}_{(1-\alpha),w}(R_3)$ respectively the VaR value of ARTO.JK, ITMG.JK, and MIKA.JK. $VaR^{CFE}_{(1-\alpha),w}(S_3)$ would be the upper bound value of the VaR portfolio P_3. The following parameters will be used to calculate the upper bound of VaR using the CFE approach:

Table 6. VaR-CFE Parameters for ARTO.JK, ITMG.JK, and MIKA.JK

Parameters		**Value**		
		ARTO.JK	**ITMG.JK**	**MIKA.JK**
Excess kurtosis		0	0	0
Sig level (1-α)		95% & 99%	95% & 99%	95% & 99%
Cornish-Fisher Expansion value (CFE Value)	α = 1%	-2.868	-2.713	-2.711
	α = 5%	-1.619	-1.632	-1.378
Holding period (day)		1, 3, and 5	1, 3, and 5	1, 3, and 5

Soruce: Research finding.

Since the observation period ended on 10/21/22, this date was the reference in determining risk predictions for the following periods. By using equation (23), the VaR- CFE value for a single asset at several confidence levels and holding periods is as follows:

Tabel 7. VaR- CFE for a Single Asset at 95% Confidence Level

	Holding period		
	1 day	**3 days**	**5 days**
ARTO.JK	-0.0588	-0.1018	-0.1315
ITMG.JK	-0.0480	-0.0831	-0.1073
MIKA.JK	-0.0326	-0.0564	-0.0729

Soruce: Research finding.

At the 95% confidence level, the single asset with the most significant loss risk was ARTO.JK, during the 1-day holding period, the VaR- CFE prediction was -0.0588, meaning that the value of the loss risk that would occur for one period after 10/21/22 was 5.88 % of the total funds invested. If an investor had IDR 100 million in funds, then within one day after that, the maximum possible loss that the investor would experience was IDR 5.88 million. Then, for a holding period of 3 and 5 days, the predicted loss of VaR- CFE for ARTO.JK was 10.18% and 13.15%, respectively. Furthermore, the single asset with the smallest loss was MIKA.JK. This company engaged in the health sector had a predicted loss of 3.26% for a 1-day holding period. Then, for the 3 and 5-day holding periods, the predicted loss risk was 5.64% and 7.29%. Furthermore, VaR- CFE predictions for the 99% confidence level can be seen in the following table:

Tabel 8. VaR- CFE for a Single Asset at 99% Confidence Level

	Holding period		
	1 day	**3 days**	**5 days**
ARTO.JK	-0.1074	-0.1860	-0.2401
ITMG.JK	-0.0782	-0.1355	-0.1749
MIKA.JK	-0.0631	-0.1093	-0.1411

Soruce: Research finding.

At the 99% confidence level, the stock with the highest loss prediction value and the slightest loss value was the same as at the 95% confidence level. ARTO.JK was the stock with the highest prediction of loss, and MIKA.JK was the stock with the minor prediction of loss. After obtaining the loss risk value of a single asset, then we used these results to obtain the upper bound of the VaR-ECF value for a stock portfolio made up of 3 single assets. The upper bound of the VaR-ECF portfolio was obtained by adding up the value of the loss risk of a single asset at the corresponding confidence level and holding period. The following are the upper bound of the portfolio VaR-ECF value at the 95% and 99% confidence levels:

Tabel 9. The Upper bound of Portfolio Loss Risk Using VaR- CFE

Confidence level	**Holding period**		
	1 day	**3 days**	**5 days**
95%	-0.1394	-0.2414	-0.3116
99%	-0.2487	-0.4307	-0.5560

Soruce: Research finding.

Table 9 shows that, at a 95% confidence level and 1 day holding period, the value of the loss investors received from the P_3 portfolio for the next period from 10/21/22 would not exceed 13.94%. For a 99% confidence level, the upper bound value obtained was 24.87%. As a validation, we constructed a P_3 loss prediction value to ensure that at the appropriate level of confidence, the predicted loss risk value did not exceed the upper bound value. For the 95% and 99% confidence levels, the VaR-ECF values for the stock portfolio were -0.0731 and -0.1112. This predicted value was smaller than the upper bound value, so the upper bound value obtained was valid for measuring the value of the loss risk in the stock portfolio.

Backtesting Test

In the risk prediction, the risk prediction results could be interpreted as an estimate of the losses that would occur; in other words, the risk value that would occur in these stocks was around the predicted value obtained. Since after calculating the upper bound of VaR, backtesting was carried out; thus, before making predictions, we first determined the estimation window (K_E) and test window (K_U). The lengths of both were set equal to the in-sample and out-sample periods. For more details, the distribution of the estimation window and the test window can be seen in Table 10.

Table 10. The Distribution of the Estimation Window and Test Window

Estimation Window		Test Window
k	***k* + *K***	

Estimation Window		Test Window
k	$k + K_E$ - 1	Adj-ES ($k + K_E$)
1 (10/25/21)	08/27/22	Upper bound of VaR for 08/29/22
2 (10/26/21)	(08/29/22)	Upper bound of VaR for 08/30/22
3 (10/27/21)	(08/30/22)	Upper bound of VaR for 08/31/22
…	…	…
39 (8/29/22)	(10/19/22)	Upper bound of VaR for 10/20/22
40 (8/30/22)	(10/20/22)	Upper bound of VaR for 10/21/22

Soruce: Research finding.

At the 95% significance level (α = 5%) and one day holding period, the results of the prediction of the risk of loss in the test window are as follows:

Table 11. The Upper Bound of VaR-CFE at Test Window Periods

Period	Actual Portfolio Return	Upper bound of VaR
08/29/22	-0.02992	-0.17401
08/30/22	0.02174	-0.16181
08/31/22	0.04209	-0.16412
…	…	…
10/20/22	-0.00439	-0.16321
10/21/22	0.03465	-0.1394

Soruce: Research finding.

The main concept of the backtesting test is to measure the value of the Violation Ratio, calculated based on the values of m_0 and η. The calculation of the Adj-ES violation ratio at the 95% confidence level was simulated on several values of estimated violation probability, namely 0.1%, 0.5%, 1%, 2%, 3%, 4% and 5%. With the help of software R 3.3.2, the obtained value of the ratio of violations is as follows:

Table 12. Violation Ratio Value on Several m_0 Value

m_0	Violation Ratio	m_0	Violation Ratio
0.1%	0.002	2%	0.01
0.5%	0.003	3%	0
1%	0.007	5%	0

Soruce: Research finding.

In estimating the upper bound of loss risk using VaR on stock portfolio with a 95% confidence level, the VaR could provide accurate prediction results for each violation probability VR value < 1. The value of VaR could be interpreted as predicting losses in the corresponding period. Then, the predicted upper bound of VaR-CFE was applied to the following 16 periods after 10/21/22.

Table 13. The Predicted Upper Bound of VaR-CFE

Period	Predict	Period	Predict
10/22/22	-0.1421	10/30/22	-0.1566
10/23/22	-0.1467	10/31/22	-0.1533
10/24/22	-0.1527	11/1/22	-0.1520
10/25/22	-0.1522	11/2/22	-0.1713
10/26/22	-0.1387	11/3/22	-0.1515
10/27/22	-0.1417	11/4/22	-0.1454
10/28/22	-0.1442	11/5/22	-0.1662
10/29/22	-0.1412	11/6/22	-0.1398

Soruce: Research finding.

The highest estimated loss occurred on 11/2/22, which was -0.1713. Thus, investors needed to prepare a good risk management strategy so that the losses did not have a destructive impact on the investment being carried out. During this period, investors were not advised to sell their assets on the Indonesia Stock Exchange because when there was a significant loss, the asset price decreases, increasing the loss if investors sell their assets.

5. Conclussion

Based on the analysis in section 4, we conclude that the three single assets we have chosen to compile the portfolio follow the optimal portfolio rule because the correlation between single assets is negative. Based on the comotonic properties, the portfolio loss risk ceiling is equal to the sum of the loss values of every single asset. In this study, the loss risk is measured using the VaR-CFE model. The stock with the highest loss value for a single asset is ARTO.JK. At the 95% confidence level and one day holding period, the predicted value of ARTO.JK's stock price loss for the period after 10/21/22 is -0.0588. Furthermore, the single asset with the smallest predicted loss value is MIKA.JK operates in the health sector. For a 95% confidence level and a one-day holding period, the loss risk for this stock is -0.0326. Finally, the upper bound for portfolio loss risk at the 95% confidence level is -0.1394. After validation, the results show that the VaR-ECF value of the stock portfolio at the 95% confidence level and one-day holding period is -0.0731; this value is proven to be lower than the upper bound predicted value.

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