Iranian Economic Review, Vol.9, No.10, Spring 2004

Economic Growth as a Nonlinear and Discontinuous Process

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Abstract

Structural changes were the main focus of many studies in recent years. Changes which alter the socio-economic status of a nation from deprived and traditional nature to a developed and modern one. The main hypothesis of this study is that, the economic development does not occur through a regular flow of life (linear fashion), and without resilience, but it certainly needs to pass through a momentum and exceed a minimum threshold. We apply catastrophe and other theories to show the nonlinearity and jump in the economic growth path.

Empirical results of the catastrophe models imply that sudden jumps and nonlinearity is a case for NICs economic growth. The CUSP model is a superior one among the other catastrophe models which verifies the nonlinearity of the growth path. In addition to more than 500 regressions with different control variables in each regression for individual countries, some panel regression for all countries data confirms validity of individual results. Chaos models in logistic form results in negative Lyapunov exponent but not significant statistically which can't be reliable. However, Lyapunov exponent estimation by means of Matlab gives negative exponent that are reliable for some of the countries. Terasvirta and White nonlinearity test reject linearity in growth path for Singapore and Thailand. Also jump test agree with our hypothesis which involve with existence of sudden jumps in growth path for all of NICs.

Having these results, the policy prescriptions for LDCs could be; targeting nonlinear growth path in development plans, and appealing to foreign resources as well as constructive changes in institutions, polity and incentive structure.

Keywords: Nonlinear Growth, Catastrophe Theory, Discontinuities, Neural Networks, Chaos Theory.

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I- Introduction

Many studies have attempted to determine the key factors of economic growth and its changes. Although economic growth is no longer treated as the only determinant factor and indices of economic development, however it is still considered as one of its most important components. In most traditional growth models, capital and labor are taken as exogenous variables, and technical progress is the core element of the so called neoclassical models. But institutional aspects of economic policies which have essential role in understanding the relation between growth and development are often neglected from the growth models. Recently, the role of institution has been emphasized again by Mancur Olson (1996). Olson like Hirschman (1958) explained how resources are utilized rather than their quantitative role. Other economists who are engaged in development of growth models introduced the endogenous growth models that extended the list of variables included in the models. Islam (1995) and Mankiw et al. (1992) have studied country specific effects on growth. Also Chenery and Syrquin (1975) and Chenery (1989) focused on initial condition of growth. Some economic historian raised the issue of catch-up problem and reached to new points in growth theory. Abramowitz (1986), for example, argued that the differences in the catch- up growth reflects the "social capability" of which the standard of educations an important component. Benhabib and Spiegel (1994) and Islam's (1995) papers support the results of Abramowitz's formulations. Social capability which Abramowitz points out involves institutions and incentive structures. Various Variables such as education, price distortions, inflation, macroeconomic instability, asymmetric information, contract enforcement, rent seeking, property rights, political, legal and financial infrastructures are proposed by many economists for explaining the residual of growth. Syrquin (1998) adds to this list by emphasizing on "structural changes' in the growth phenomenon. Syrquin's expression is reflected best in the following statement: "structural change is at the center of economic growth".

Also, Kuznets (1971, p.348) believes that economic growth is impossible without structural change. He believes structural changes occure not only in the economy but also in the institutions and even in the ideas. Notwithstanding, the economists have not entered the structural variables as explanatory variables in the growth regression models.

In this paper, the first development is about the entering and testing the significance of the industry share variable of value added in the growth model. Nonlinearity test and resilience in the growth path are among important aspects that will be dealt with in this paper. Rostow's take-off is the first claim on nonlinearity in growth path (Rostow 1960). He believes that, pioneer industries and prevalent technical penetration in basic industries create a phenomenon that has a resilient characteristic and commence a new sphere of economic life which can be called "economic take-off." Morris & Adelman (1997, p.883) say, "The process of economic development is strictly nonlinear and multidimensional." Schumpeter (1934) claims, "economic development is spontaneous and discontinuous change in the channels of the circular flow and these disturbances of the center of equilibrium appear in the sphere of industrial and commercial life".

These are a few examples which economists have proposed as nonlinearity of the growth path. Of course, in the experimental studies, nonlinearity test of growth path has not gained any attention.

The main hypothesis of this study is that, the economic development does not occur through a regular flow of life (linear fashion), and without resilience, but it certainly needs to path through a momentum and exceed a certain threshold. Thus, growth is studied not only as a nonlinear phenomenon but also as a discontinuous one. The nonlinearity of the systems can be verified by several ways. We may apply catastrophe theory to show the nonlinearity and jump in the economic growth. The crucial point in studying growth from catastrophe theory is the spontaneous or sudden change of growth because of changing in some of the control variables. Besides catastrophe theory, other nonlinearity tests are used.

Next sections of this paper are organized as follow: Section 2 elaborates the theorical foundations of catastrophe theory and its application in economics. Section 3 identifies the collection and processing of data, analytical tools and estimation processes. The results of regression analysis and their interpretation are discussed in the fourth section, and the final section is devoted to the conclusion and suggestions for future studies.

II- Catastrophe Theory and Applications to Economics

Catastrophe theory (CT hereafter) is a method for studding of structural changes of dynamical systems. Many phenomena in the world can be modeled by differential equations that are continuous and change smoothly between equilibriums. However in real world one can see frequently discontinuous and sudden changes in structure of dynamical system's topological characteristics (Zeeman 1975). These changes not only are important from mathematical point of view but also in applied works. Sudden jumps can transfer the system from one kind of behavior to the other one. In applied sense CT is a theory for study of sudden jumps in real world. Rene Thom (1972) show that nonlinear potential functions which are higher than 3 in degree (cubic and higher) can show structural instability. Meaning of the structural instability is change of the topological features. Also he shows that all of the systems that has less than four control variables (parameter) can be stated by seven elementary catastrophe equations.

This statement is named as Thom classification theorem. The number of catastrophe equation rose to 25 by Russian mathematician V. Arnold. However, seven elementary equations are sufficient in practical works. Table1 shows these equations that one of them "Cusp "is the most famous one. We will debate the first and the second one in more detail.

Table 1: Catastrophe Elementary Equations

Function	Behavior dimension	Control dimension	Name	Type
$x_1^3+\tau_1x_1$	1	1	Fold	Cuspoid
$\pm(x_1^4+\tau_2x_1^2+\tau_1x_1)$	1	2	Cusp	
$x_1^5+\tau_3x_1^3+\tau_2x_1^2+\tau_1x_1$	1	3	Swallowtail	
$\pm(x_1^6\tau_4x_1^4+\tau_3x_1^3+\tau_2x_1^2+\tau_1x_1)$	1	4	Butterfly	
$x_1^7+\tau_5x_1^5+\tau_4x_1^4+\tau_3x_1^3+\tau_2x_1^2+\tau_1x_1$	1	5	Wigwam	
$x_1^2+x_2-x_2^3+\tau_3x_1^2+\tau_2x_2+\tau_1x_1$	2	3	Elliptic	Umbilic
$x_1^2x_2+x_2^3+\tau_3x_1^2+\tau_2x_2+\tau_1x_1$	2	3	Hyperbolic	
$\pm(x_1^2x_2+x_2^3+\tau_3x_1^2+\tau_2x_2+\tau_1x_1)$	2	4	Parabolic	
$x_1^2x_2+x_2^5+\tau_5x_2^3+\tau_4x_2^2+\tau_3x_1^2+\tau_2x_2$	2	5	Second hyperbolic	
$x_1^2x_2+x_2^5+\tau_5x_2^3+\tau_4x_2^2+\tau_3x_1^2+\tau_2x_2$	2	5	Second hyperbolic	
$\pm(x_1^3+x_2^4+\tau_5x_2^2+\tau_4x_2^2+\tau_3x_1x_2+\tau_2x_2$	2	5	Symbolic	

Source: Poston & Stewart 1976, p.74

Let $\dot{X}=\frac{dx}{dt}=f(\vec{x},\vec{\tau})$ be a dynamical system where X is vector of state variables and τ is vector of control parameters. Smooth change of parameters usually leads to smooth change in X that is state of the system in time t .However, in some cases which smooth change of a control variable drives the system on discontinuous path the catastrophe is realized. As a simple example supposes:

$$v(x,\tau) = \frac{x^3}{3} + \tau x \tag{1}$$

$$\frac{dV(x,\tau)}{dx} = x^2 + \tau \tag{2}$$

For the singularities let:

$$x^2 + \tau = 0 \tag{3}$$
$$x^2 = -\tau$$

The value of τ is very important. For $\tau < 0$ system has two equilibrium (i.e. $\pm\sqrt{-\tau}$). But if $\tau > 0$ system hasn't any equilibrium. In other words when parameter of the system (τ) smoothly change from negative values to the positives, in the zero, qualitative features of the system changes fundamentally. Strictly speaking zero point is a catastrophe of the equation. This equation called "Fold" that has one state and one control variable. A system with one state and two control variables can be formulized by "Cusp" catastrophe model. The general form for Cusp is:

$$V(x,\tau_1,\tau_2) = \frac{1}{4}x^4 + \frac{1}{2}x^2 + \tau_1 x \tag{4}$$

$$\frac{dV(x,\tau_1,\tau_2)}{dsx} = x^3 + \tau_2 x + \tau_1 \tag{5}$$

A typical Cusp model with economic variables is depicted in FIG.1. For the first time in economics Zeeman (Zeeman1974) used Cusp for effect of battle between fundamentalists and chartists on DJ index in the stock market. In that study state variable was DJ index and control variable were chartists and fundamentalists shares of stock market trades.

This model can be shown by FIG.1. Instead of growth will be DJ and control variables will be chartists' and fundamentalists' market share. Another example in Rosser (1991) is related to OPEC and oil market price changes. Various examples in different areas confirm flexibility of cusp type catastrophe model. Poston and Stewarts (1978) give some interesting examples for applications of the cusp model.

In economics discontinuities and sudden changes are not rare and "structural change" is used at least in three fields of Econometrics, Development Economics and Planning. In the first one structural change is used for change of parameters of an equation . The second field, it coined for changes in relative importance (value added and employment shares of sectors in GDP and total labor force respectively) of the sectors of an economy. Planning literature applies the term "structural change" for telling about change of technical coefficients of Leontief input - output matrix. Application of these terms in economics is irrelative to the term "structural change" in differential topology, at least in historical manner. However, there are various subjects that can be related to the CT from theoretical point of view. In Puu (1991, P.57) transition of agrarian economy to modern industrial economy described by following expression:

There are nomadic, agrarian and industrial
equilibria, but some impact is needed to
Move system from one state from one to another.

Also, about change of parameters of production function and its effects on equlibria Puu (1991, P.56) says:

... Making an attained equilibria suddenly
Evaporate so that the system must take
Off to a new one.

The word of evaporate is used very properly for explanation of the sudden jumps. One of the recent works with empirical content is capital formation in centralized planning economies. (Rosser et.al. 2001). Application of CT in business cycle is a nice work from H. Varian (1981) which is a theoretical one without estimation and focused on cusp type catastrophe model. Also there are some works about Philips curve (Fischer and Jemmernegg. 1986), multimodal density functions estimation (Cobb 1982, 1985, 1998), human affaires and psychology (Guastello 1995, 2000) and behavioral and biological sciences (Zeeman 1975, 1974, 1977) among the others .

In formulation of economic phenomena by means of cusp catastrophe model, control variables can be institutional change, technological change, cultural change, external shocks (such as huge rainfall in agrarian economies), foreign aid, foreign direct investment, export, industrial exports, R&D expenditure, and some other socio - economic variables are among the most susceptible proxies for control variables . In departure from one equilibrium to another one (e.g. from instable growth to the stable one) as noted before, big shocks that cause sudden changes are needed. The GDP growth rate is the state variable in this paper .In addition to division of variables in to two kind (state and control variables) CT literature categorize variables to slow and fast ones (Varian 1981). Slow variables can be proxies for control variables, but fast variable is the state or dependent variable. Classification of variables in to two categories (fast and slow) is a relative classification and depends on the context.

In other words a variable that is fast in one system may be slow in the other system. Structural change, for example, is a slow variable when is compared with institutional changes that may need time as long as a millennium in Williamson's words (Williamson, 2000). In the next section we will debate cases which control variables are not determined by theoretical considerations.

The second step in formulation of catastrophe models for growth is determination of functional form of potential function. If theory specifies control variables exactly the functional form that should be used is determined automatically. Because if one find out that only one control variable cause sudden jumps in the growth path, then the potential function is "Fold" . The difficulty in` this method is lack of any theoretical information. In these cases cusp is a proper choice because of its interesting properties namely, “Bimodality”, “Sudden Jump”, “Hystery”, "Inaccessibility" in a portions of the surface and "Divergence".

The compatible phenomenon with these properties can be modeled by cusp catastrophe. In cusp modeling one implicitly assumes that phenomenon under study are dynamical systems that are compatible with two almost conflict institutional order such as "market" and "planed" economy that may be attained to combination of them also . But when one state of the system imply destruction of another state (combination of them is not possible) "Fold" is the proper catastrophe model. Swallowtail model can be proposed for unsuccessful tries to transfer from one equilibrium to another one, which has three control parameters. The third world experience in the economic growth domain may be the case for this type of catastrophe model. Because less developed countries didn’t escape from low and unstable economic growth rate. Whereas NICS pass this stage successfully and their growth dynamics most likely can be captured by means of “cusp “ model.

In the systems with "Source - Message – Receiver" nature; moves from states to each other may be modeled with a four control variable model namely "Butterfly". One of the best examples of this model is technological imitation that third world countries (Receivers) passively imitate technological orders (Message) of advanced countries innovations (Sources). Umbilic (Hyperbolic, Parabolic and Elliptic) models are appropriate for cause and effect models. These models have two states and 3-5 control variables that are simultaneous equations in econometric sense.

General discussions such as the above mentioned one can be applied in choosing appropriate model partly, but remaining elaborations in model selection translated to empirical concepts and procedures. Statistical and theoretical significance of results will help to model builder in the selection of the proper model. Significance of coefficients both statistically and theoretically is a little different from standard rules that will be debated in the next section.

III- Data and Method

Data set for dependent variable growth and independent variables labor force, capital as usual regressors; and structural changes, institutional factors, big shock factors, etc. as control parameters obtained from World Bank CD titled WDI 2002. Annual growth rate of the real GDP is the dependent variable and considered over 41 years (1960 - 2000) for NICs. Independent variables such as labor and capital are important variables that drive growth on a smooth path. Other variables listed in table 2 are among the variables skeptical for source of sudden jumps in growth. Institutional changes are slow variables that can cause sudden jumps and discontinuities in the growth path. There is not variable that measure institutional changes directly therefore some variables such as news papers per 1 000 people, literacy rate etc., selected from 575 series which are given in the WDI. For technological changes that make discontinuities in growth path, R& D expenditures, scientists and technicians employed in R&D seem more relevant than the others.

Structural changes can be measured by value added of the primary and secondary sectors as a percent of GDP. Structural changes enforce growth if it be relatively fast and robust (Syrquin1998, P.158). The variables industry value added as a percent of GDP and manufacturing as a percent of GDP are the proxies for structural changes.

Some other variables such as foreign direct investment and foreign aids supposed give big shocks or big push (in word's of Rosenstein - Ruddan) to economy and boost it in development path.

Now we have variables that their number amounts to 25 or so and theory doesn't shade light on selection of variables for catastrophe models which can use limited number of control variables. (At most four control variable). Therefore, we will adopt following strategies:

Let variables in the models in turn and pick one that is the best statistically and theoretically.

Clustering the variables to several categories and extract two first factors by factor analysis.

We don't intend explain cluster and factor analysis here. However, pointing to some points can help to explanation of procedure of our work. When one isn't able to classify variables by theory she can use one of the clustering methods. In catastrophe theory and model building one can use clustering for classifying the variables as fast and slow one, and control variables of some kind in one cluster, the other kind in the second one and so forth. After clustering, factor analysis will extract factor with largest variance and lead to get the given number of variables that are proxies for control variables. Rotation of factors is also interesting in this context. Because when we need two control variables.

Table 2: Independent variables definition and classification

Variants and definitions	Availability (years)	Name of independent variable	Type
GDP per capita (constant 1995 US$)Gross capital formation (% of GDP) Gross domestic savings (% of GDP) Gross fixed capital formation (% of GDP)	Between 30-40	1. Labor force 2. capital stock 3.gdp per capita 4.gross domestic saving	Normal
Agriculture, value added (% of GDP) Agriculture, value added (annual % growth) also for other series in this category Manufactures exports (% of merchandise exports)	Between 35-40 except Hong Kong	1. Agricultural value added 2. Industry value added 3. Manufacturing value added 4. Agricultural value added growth 5. Industry value added growth 6. manufacturing value added growth	Structural
Aid (% of central government expenditures) Aid (% of gross capital formation) Foreign direct investment, net inflows (% of GDP) Foreign direct investment, net inflows (% of gross capital formation)	Less than 30	1. Foreign aids 2. Foreign direct investments 3. Capital inflow	External
High-technology exports (% of manufactured exports) Research and development expenditure (% of GNI) Scientists and engineers in R&D (per million people) Labor force, total Technicians in R&D Information and communication technology expenditure (% of GDP)	For some series less than 15	1. R&D Expenditure 2. number of scientists and technicians in R&D 3.high-tech export	Technological
Illiteracy rate, adult total (% of people ages 15 and above	For news paper at most 12	1. News papers per 1000 2. Illiteracy rate	Institutional

In the above figure points 1 to 6 are variables that are all in the positive region. But after rotation Control will compose of negative and positive Variables.

With two opposite effect, it will be obtained by proper rotation when even they don't satisfy this condition before rotation (Figure 2). Also factors are very useful in model building with unobservable or complex variables that are not readily available. These factors are latent variables that are familiar in econometric literature. Interpretation of principal components and factors isn't straightforward, but in some case we aren't interested in "what cause an effect", and only existence of a relation (such as VAR models) is sufficient. In some cases variables itself and individually aren't important but in common can extract synergetic effects that make them very important. Institutions individually may have little effect but common effect of them can't be denied. Factor analysis can be one of the powerful approaches in these cases.

In spite of above mentioned strengths of multivariate statistical analysis there is some shortcomings that limit their usefulness. One of them is incomplete data. For example in our data set, observations for some variables such as R&D expenditure is available only for some years and other variables also aren't so long. This may lead us to unstable and contradicting results. Therefore we use factor and cluster analysis only for variables with relatively complete range of observations. Software such as Minitab 13, Statistica and S_plus 2000 can be used for this end and methods are debated "Multivariate statistical Analysis" text books e.g. Mardia et.al. (1979). Results of cluster and factor analysis are given in table 3.

Table 3: Dependent and independent variables clusters

CONTRY NAME	CLUSTER1	CLUSTER2	CLUSTER3	CLUSTER4	CLUSTER5
Indonesia	Agva,ilra Aidcap,aidg	Aggr,gdpgr ,ivagr,mvagr	Capexp,fdigcf fdigd		Gdpper,mex,mva, Gensave,iva
Korea	Aggr,capexp	Agva,aidcap, Aidg,ilra	Gdpgr,ivagr mvagr	Fdigcf,fdigd	Gdpper,mex,iva, Mva,gensav
Malaysia	aggr, agva ilra,aidcap aidg,capexp	capexp	gdpgr,mvagr ivagr	Fdigcf Fdigd	Gdpper,mex. Mva,iva,gensav
Singapore	Aggr,	Agva,ilra aidcap,aidg	Gdpgr,ivgr mvagr	Iva,mva	Fdigcf,fdigd Capexp,gdpper Mex,gensave
Thailand	Agva,ilra Aidcap,aidg	Aggr,gdpgr Ivgr,mvagr	Capexp,fdigcf fdigd	Gensav,sgdpper Iva.mex,mva	

As one can see in the table 2, there are few observations for some variables. Therefore, after estimation regressions for individual time series of each NICs regression with panel data will run for all of the NICs. In this case problem of data frequency can be solved partly.

Catastrophe models significance imply nonlinear growth path in a special type, but rejection of catastrophe models isn't rejection of nonlinearity. Because short data range, unavailability proper control variables (Lack of good censuses system) or unsubstantial discontinues all are the factors that will affect results. When discontinuities are small, methods such as chaos can be aspired, and when nonlinearity is not in special form neural networks, works well than the others.

Table 4: Methods of studding nonlinear phenomena

Nam of method / theory	Application	Further consideration
Catastrophe	Discontinuity Nonlinearity Bifurcation	This theory needs for determination of control variables and without of them is not applicable but, when one determines control it is better than the others and also abilities that other methods don't have
Chaos	Un substantial Discontinuities Forecast bility Nonlinearity	This theory can be applied for the variables or series silently (without control variables) Fixed points can be studied via this theory for some areas.
Neural networks	Nonlinearity test Forecasting	This method is strong when aim is forecasting without any assumption about form of nonlinearity. But it needs large samples.

Some applications and strength of these methods are listed in table 3.

After e stimation o f c atastrophe m odels, n onlinearity and d iscontinuity o f growth will be tested by neural networks and chaos models. However because of space saving in this paper we don't pay to theoretical contents of these theories. The most relevant work to ours are Guastello (1995), Rosser (1991, 1999), Terasverta (1993), Mills (1999), Hesia (1991), Frances (2000) .

In the next section operational steps and concepts in regression forms and results will be presented.

IV- Estimation and Results

In economics estimation and testing of hypothesis is a dominant procedure at least in empirical works. Models such as catastrophe and chaos, but not neural network, are deterministic models. Statistical version of catastrophe models is result of nice works of various branches of sciences such as mathematics, statistics, physics, psychology, economics, computer science, politics, etc. However there are some works among the other, such as Zeeman (1976) Cobb (1982, 1998), Gustello (1995, 2000) Rosser (1999), Fischer & Jammernegg (1986). The regression specifications can be summarized as follows:

Cusp Type I

$$D(z_t) = \alpha + \beta_1 z_{t-1}^3 + \beta_2 z_{t-1}^2 + \beta_3 cont_{1t-1}.z_{t-1} + \beta_4 cont_{2t-1} + \varepsilon_t$$

Where z_t is GDP Gross Domestic Product that is standardized for location and scale by subtracting minimum of observations and dividing by standard deviation, $cont_1$and $cont_2$ are control variables that stands for variables listed in table 2. Replacement of these variables in turn will lead to estimation of 91 regressions with different control variables. For other types of catastrophe models procedure is the same.

Cusp type II

$$D(z_t) = \alpha + \beta_1 z_{t-1}^3 + \beta_2 z_{t-1}^2 + \beta_3 cont_{1t-1}.z_{t-1} + \beta_4 cont_{2t-1} + \beta_5 cont_{1t-1} + \varepsilon_t$$

Cusp type III

$$D(z_t) = \alpha + \beta_1 z_{t-1}^3 + \beta_2 z_{t-1}^2 + \beta_3 cont_{1t-1}.z_{t-1} + \beta_4 cont_{2t-1} + \beta_5 cont_{1t-1} + \beta_6 cont_{2t-1} z_{t-1} + \varepsilon_t$$

For cusp type II and III such as I, 91 regressions estimated and according to $\overline{R}^2$ criteria optimal model selected.

In order to further examination and avoiding prejudgment in the selection of catastrophe equation following models is fitted and will be compared with the cusp models results for selection of the best model.

Fold

$$D(z_t) = \alpha + \beta_1 z_{t-1}^2 + \beta_2 cont_{1,t-1} + \beta_3 z_{t-1} + \varepsilon_t$$

Swallowtail

$$D(z_t) = \alpha + \beta_1 z_{t-1}^4 + \beta_2 z_{t-1}^3 + \beta_3 z_{t-1}^2 cont_{1,t-1} + \beta_4 z_{t-1} cont_{2,t-1} + \beta_5 cont_{2,t-1} + \beta_6 z_{t-1} + \varepsilon_t$$

Butterfly

$$D(z_t) = \alpha + \beta_1 z_{t-1}^5 + \beta_2 z_{t-1}^4 + \beta_3 z_{t-1}^3 cont_{1,t-1} + \beta_4 z_{t-1}^2 cont_{2,t-1} + \beta_5 z_{t-1} cont_{3.t-1} + \beta_6 cont_{4.t-1} + \beta_7 z_{t-1} + \varepsilon_t$$

Also umbilic models that are simultaneous equations in econometrics terminology can be showed as follows:

$$D(z_{1t}) = \beta_1 + \beta_{12} z_{1,t-1}^2 + \beta_{13} cont_{1,t-1} + \beta_{14} cont_{3,t-1} z_{2,t-1} + \varepsilon_{1t}$$
$$D(z_{2t}) = \beta_2 + \beta_{22} z_{2,t-1}^2 + \beta_{23} cont_{2,t-1} + \beta_{24} cont_{3,t-1} z_{1,t-1} + \varepsilon_{2t}$$

In this model there are two state and three control variables. States can be agriculture and industry value added and control variables are the same to the cusp one. The control variables $cont_1$And $cont_2$ are specific to equation one and two respectively but $cont_3$ is common between them. Limited period for available data and small sample concerns induce us to apply the models for panel data in addition to individual countries regressions. In our panel cross identifiers are Indonesia, Korea, Malaysia, Singapore and Thailand. Also estimation method will be weighted fixed effects because of exceeding number of parameters than the cross identifiers.

Chaotic models in logistic form and also difference equation form can be used for examining existence of chaotic behavior and nonlinearity. According to Guastello (1995) equations for this aim are.

$$Z_t = \alpha + \beta z_{t-1}(1 - z_{t-1}) + \varepsilon_t$$

$$Z_t = \alpha + \beta_1 e^{\beta_2 z_{t-1}} + \varepsilon_t$$

β_2 Is lyapunov exponent and its sign is critical for determination of chaos. The positive lyapunov exponent means series is chaotic and the negative one is in favor of non chaotic series. If a series has negative lyapunov exponent, it will be converge to some fixed points. Dimension of time series and dynamical systems can be determined by $D = e^{\beta_2}$ which is special case of following equation:

$$z_t = \alpha + e^{\beta_1 z_{t-1}} + e^{\beta_2 z_{t-2}} + \cdots + e^{\beta_k z_{t-k}} + \varepsilon_t$$

And

$$D = e^{\beta_1 + \beta_2 + \beta_3 + \cdots + \beta_k} - 1$$

Also, some nonlinearity tests based on neural networks can be used for our time series that Terasvirta (1993). This test is preferable to other versions such as white ANN test because of its superiority in small samples (Frances 2000). Therefore, we will use Terasvirta test for examination nonlinearity of economic growth. Regression model of this test is as follows:

$$Z_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^2 + \beta_4 Z_{t-1} Z_{t-2} + \beta_5 Z_{t-2}^2 + \beta_6 Z_{t-1} Z_{t-2} Z_{t-3}$$
$$+ \beta_7 Z_{t-1}^2 Z_{t-2} Z_{t-3} + \beta_8 Z_{t-1} Z_{t-2}^2 Z_{t-3} + \beta_9 Z_{t-1} Z_{t-2} Z_{t-3}^2$$

One of the assumptions of this test is stationarity that will be satisfied for growth time series.

Before running programs for estimation over 500 regressions with various control variables we clustered variables and derived some factors from each cluster. Also we got factors from all of independent variables which could be used as control variables. Table 4 shows clusters for countries. In spite of their theoretical attractions factor and cluster analysis couldn't give control variables stronger than the variables themselves.

Some programs in Eviews programming formats wrote and ran which their results are given in table 5. Results imply that overall significance of cusp model isn't rejected for all of NICs. Of course a lot missing observations for Hong Kong failed running of programs and, results are available only for five countries. (Indonesia, Malaysia, Korea, Singapore and Thailand). Empirical considerations confirm literature of nonlinear growth and imply that NICs experienced some jumps in the 1960 - 2000. Detected control variables are different for five countries. Cusp models for all of countries are superior to other catastrophe models (Fold, Swallowtail, Butterfly and umbilic).

Insignificance of factors can steam from low number of observation that gives unstable factor loadings and using unrelated factors. Finding proper rotation method is not easy task and because of very extend nature of this work is not possible practically. Therefore, we used only unrelated factors. Factor and cluster analysis is done by means of Minitab11 and regressions ran by Eviews 3.1.

Table 5: Catastrophe Models Estimation Results

Country	Number Regression	Best Model : coefficients and t - stats	f/prom	Considerations
Indonesia	273,cusp	$D(z)=\underset{(9.12)}{15.76}-\underset{(-2.52)}{0.19Z^3}+\underset{(3.07)}{2.11Z^2}$ $-\underset{(-10.25)}{1.69X_7}-\underset{(-10)}{9.91X_8}+\underset{(8.33)}{2.1X_8Z}$ $+\underset{(11.83)}{8.3X_7}-\underset{(-5.57)}{9.3Z}$ $\overline{R}^2=81.67$	18.19	is rejected H_0 cusp confirmed controls $x_7$andx_8
Thailand	273,cusp	$D(z)=\underset{(6.7)}{5.8}-\underset{(-9.03)}{0.14Z^3}+\underset{(11.07)}{1.64Z^2}$ $-\underset{(-4.41)}{0.58ZX_{11}}-\underset{(-2.93)}{0.89X_3}+\underset{(2.7)}{1.97X_{11}}$ $-\underset{(-13.79)}{5.29Z}$ $\overline{R}^2=72.93$	14.02	is rejected H_0 cusp confirmed controls x_{11}andx_3
Singapore	273,cusp	$D(z)=\underset{(-2.17)}{-1.16}+\underset{(4.4)}{0.39Z^3}-\underset{(-4.29)}{2.18Z^2}$ $-\underset{(-10.25)}{0.07ZX_9}+\underset{(4.03)}{0.88X_{14}}+\underset{(3.30)}{2.4Z}$ $\overline{R}^2=51.51$	6.74	is rejected H_0 cusp confirmed controls $x_9$andx_{14}
South Korea	273,cusp	$D(z)=\underset{(4.69)}{22.38}-\underset{(-3.3)}{0.81Z^3}+\underset{(2.77)}{5.49Z^2}$ $+\underset{(3.66)}{1.32ZX_9}-\underset{(-4.25)}{16.06X_{15}}+\underset{(4.11)}{3.76X_{15}}$ $-\underset{(-3.7)}{6.3X_9}-\underset{(-2.7)}{13.86Z}$ $\overline{R}^2=71.43$	9.21	is rejected H_0 cusp confirmed controls $x_9$andx_{15}
Malaysia	273,cusp	$D(z)=\underset{(2.22)}{9.41}-\underset{(-1.92)}{0.15Z^3}+\underset{(2.2)}{1.8Z^2}$ $+\underset{(2.54)}{0.94ZX_3}+\underset{(3.04)}{0.00001X_{16}}-\underset{(-3.27)}{7.97X_3}$ $-\underset{(-2.76)}{8.57Z}$ $\overline{R}^2=0.59$	9.99	is rejected H_0 cusp confirmed controls $x_3$andx_{16}
panel	273,cusp	$D(z)=\underset{(3.02)}{0.002Z^3}+\underset{(2.98)}{0.2\ ZX_3}$ $+\underset{(1.81)}{0.41X_5}-\underset{(-3.9)}{2.14X_3}-\underset{(-7.2)}{1.44Z}$ $\overline{R}^2=42.04$	38.63	is rejected H_0 cusp confirmed

Results of panel data regressions presented in table 5 shows that cusp is confirmed as whole but cubic term is not significant.

One of the interesting results is insignificance of labor and capital growth variables. This means that:

1- NICs growth is highly debated to foreign resources such as aids and FDI.

2- Although capital and labor factors are not significant in jumps of growth and its discontinuities, however this is not in contradiction with relevance of labor and capital in growth process.

Chaos models show that laypunov exponent are negative for all of five NICs and growth isn't chaotic. Therefore, there is no dependency to initial condition in growth of NICs. The main reason is that big foreign shocks released these countries from their traditional bottlenecks. Also negative laypunov exponent gives information on trajectories of times series and systems that imply convergence to some fixed points in this case. Table 6 shows results of exponential form of logistic and nonlinearity test of Trasverta.

Nonlinearity test by means of exponential form of logistic map lead to insignificant F_stats and we couldn't compute Lyapunov exponent. However, Lyaponov exponent estimation in Matlab program by means of an algorithm [1] give negative Lyapunov exponent and lead us to conclude that GDP growth for NICs is not chaotic (FIG.3).Another test for chaos and nonlinear dependence, BDS test, can be done in Eviews 4.0.For doing this test we regressed GDPGR (GDP growth rate) on some ARMA models and after selecting of the best model the test did for residual of it. In other words the linear information contained in GDPGR series extracted by best ARMA model. Also Z-stats for BDS tests can be got from the first author. The tests were done by various starting values, different assumptions about on ε and using bootstrap option due to limited number of observations. Number of repeats was chosen 500 for all of tests. The main reason for the selecting of 500 is computational caveats, ability of Eviews and extend ness of work. Results of this test shows that growth has a nonlinear dependence and its dimension higher than 2 and in some cases is four. However BDS test isn't proper for testing of nonlinearities in general because of disability of it in detecting static nonlinearities.

Table 6: Results of Chaos and Trasvirta nonlinearity test

Test	Country	Estimated Equation	F	T-stat β_3 of	Interpretation
Chaos	Indonesia	$Z_t = 3.07 + .26 Z_{t-1} e^{0.066 Z_{t-1}}$	0.049	1.156	Lyapunov exponent is positive but insignificant
	Thailand	$Z_t = 3.08 + 0.42 Z_{t-1} e^{0.03 Z_{t-1}}$	0:002	0.803	Lyapunov exponent is positive but insignificant
	Singapore	$Z_t = 6.99 + 0.01 Z_{t-1} e^{0.23 Z_{t-1}}$	0.105	1.004	Lyapunov exponent is positive but insignificant
	South Korea	$Z_t = 7.11 + 0.013 Z_{t-1} e^{.18 Z_{t-1}}$	0.357	0.630	Lyapunov exponent is positive but insignificant
	Malaysia	$Z_t = 6.03 + 0.008 Z_{t-1} e^{0.30 Z_{t-1}}$	0.405	0.739	Lyapunov exponent is positive but insignificant
Nonlinearity	Indonesia	$Z_t = 4.98 + 0.015 Z_{t-1} Z_{t-2} Z_{t-3} - 0.001 Z_{t-1} Z_{t-2}^2 Z_{t-3}$	0.170		Nonlinearity is rejected
	Thailand	$Z_t = 10.45 + 0.288 Z_{t-1} - 2.296 Z_{t-2} + 0.529 Z_{t-1} Z_{t-2} - 0.029 Z_{t-2}^2 - 0.021 Z_{t-1} Z_{t-2} Z_{t-3} - 0.001 Z_{t-1}^2 Z_{t-2} + 0.002 Z_{t-1} Z_{t-2} Z_{t-3}^2$	0.000		Nonlinearity is not rejected
	Singapore	$Z_t = 7.43 + 0.028 Z_{t-1} Z_{t-2} - 0.021 Z_{t-1} Z_{t-2} Z_{t-3} + 0.002 Z_{t-1} Z_{t-2}^2 Z_{t-3}$	0.001		Nonlinearity is not rejected
	South Korea	$Z_t = 6.71 + 0.288 Z_{t-1} + 0.014 Z_{t-2}^2$	0.311		Nonlinearity is rejected
	Malaysia	$= 4.1 + 1.3 Z_{t-1} - 0.096 Z_{t-2}^2 - 0.014 Z_{t-1} Z_{t-2} Z_{t-3}$ $0027 Z_{t-1}^2 Z_{t-2} Z_{t-3} - 0.0006 Z_{t-1} Z_{t-2}^2 Z_{t-3}^2$	0.496		Nonlinearity is rejected

Terasvirta et al. nonlinearity test was done by different methods that some of them based on neural networks. Equations that were used are as follows:

$$Z_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2}$$

$$ress_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^2 + \beta_4 Z_{t-1} Z_{t-2} + \beta_5 Z_{t-1}^3 + \beta_6 Z_{t-1}^2 Z_{t-2} + \beta_7 Z_{t-1}^4 + \beta_8 Z_{t-1}^3 Z_{t-2}$$

$$ress_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^2 + \beta_4 Z_{t-1} Z_{t-2}$$

$$ress_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^2 + \beta_4 Z_{t-1} Z_{t-2} + \beta_5 Z_{t-2}^2$$

$$ress_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^3 + \beta_4 Z_{t-1}^2 Z_{t-2}$$

$$ress_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^2 + \beta_4 Z_{t-1} Z_{t-2} + \beta_5 Z_{t-2}^2 + \beta_6 Z_{t-1}^3 + \beta_7 Z_{t-1}^2 Z_{t-2} + \beta_8 Z_{t-1} Z_{t-2}^2 + \beta_9 Z_{t-2}^3$$

$$ress_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_6 Z_{t-1}^3 + \beta_7 Z_{t-1}^2 Z_{t-2} + \beta_8 Z_{t-1} Z_{t-2}^2 + \beta_9 Z_{t-2}^3$$

$$ress_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^2 + \beta_4 Z_{t-2}^2 + \beta_5 Z_{t-1}^3 + \beta_6 Z_{t-2}^3 + \beta_7 Z_{t-1}^4 + \beta_8 Z_{t-2}^4$$

For estimation of equations that begin with ress we estimated an AR (2) equation and get residuals of it .The equations mentioned above are AR (2), LSTAR4, LSTAR2, V2, ESTAR, V23, V3, RESET which can fined in (Trasvirta et.al. 1993, 2003).All of this models estimated and RSS of them as RSS of an unrestricted model compared with RSS of the AR (2) model as restricted model. The test of nonlinearity can be done by means of F test that compose from RSS of restricted and unrestricted models. In addition to these tests another equation in the following form estimated by backward method and ultimate F is used for decision that growth is linear or not.

$$Z_t = \beta_0 + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-1}^2 + \beta_4 Z_{t-1} Z_{t-2} + \beta_5 Z_{t-2}^2 + \beta_6 Z_{t-1} Z_{t-2} Z_{t-3} + \beta_7 Z_{t-1}^2 Z_{t-2} Z_{t-3} + \beta_8 Z_{t-1} Z_{t-2}^2 Z_{t-3} + \beta_9 Z_{t-1} Z_{t-2} Z_{t-3}^2$$

Results in table shows that nonlinearity can't be detected by this method and only in some cases is that the process is highly nonlinear this test may be in power. Also as Travirsta (etal.1993) note when the second derivatives of a nonlinear function are near zero power of the test is low. Therefore for dynamical systems with catastrophe points power of this test is limited. In addition to above method, Terasvirta and White test are done in R software[2] that nonlinearity in growth was confirmed for Singapore and Thailand only. The results of White and Terasvirta by means of R are similar and both of them confirm our results with Eviews.

As another test for jumps and discontinuities one may use a quantlet (In XPLORE terms) that is based on wavelets theory .This test confirms the sudden jumps inching of growth for all of NICs .However, for GDP growth as level jump can be detected only for Korea, Singapore and Thailand. In doing this test in XPLORE a parameter should be determined which default of software in our cases was. Results of this test in the XPLORE is printable graph, but can't saved in DOC format and we only can report the results in table format.

V- Conclusion and suggestions for further studies

Growth theory in nonlinearity is relatively rich however discontinuities are not so familiar one. Of course many economists point to discontinuities of growth theoretically but there is no serious works that pay attention to empirical test of it .Results of individual regressions for NICs in the form of catastrophe models confirm nonlinearity and discontinuities. The best model between seven elementary catastrophe models is the cusp one. In addition to individual regressions for each country a regression with panel data fitted that shows validity of earlier results. For handling of control variables selection because of countries censuses weakness cluster and factor analysis was used that didn't appear so helpful generally.

Test of chaotic behavior via exponential form of logistic form is inconclusive .Because in spite of positive Lyapunov exponent it is insignificant from statistical point of view. However Terasvirta nonlinearity test shows nonlinear behavior for Thailand and Singapore economies. For further examination of sudden jumps a Quant let of XPLORE is used which is based on wavelets theory, Results of this test confirm jumps in economic growth of this courtiers.

The results could be stronger if there are good proxies for institutional and technological variables. Also limited number of observations affected our results which can be elevated in future times by including longer period's observations.

Notes

1- The lyapunov exponent algorithm is from Jay Swope which was converted from a fortran version B of Wolf, Alan; Jack B. Swift, Harry L. Swinney, and John A. Vastano; "Determining Lyapunov Exponents from a Time Series," Physica 16D, pp285-317, 1985. jswope@sol.otago.ac.nz.

2- Results of nonlinearity test in R available from the first author upon request. However, R is a freeware in www.ststistics.com and interested reader can try it itself easily.

Figure1: Cusp model of Economic growth

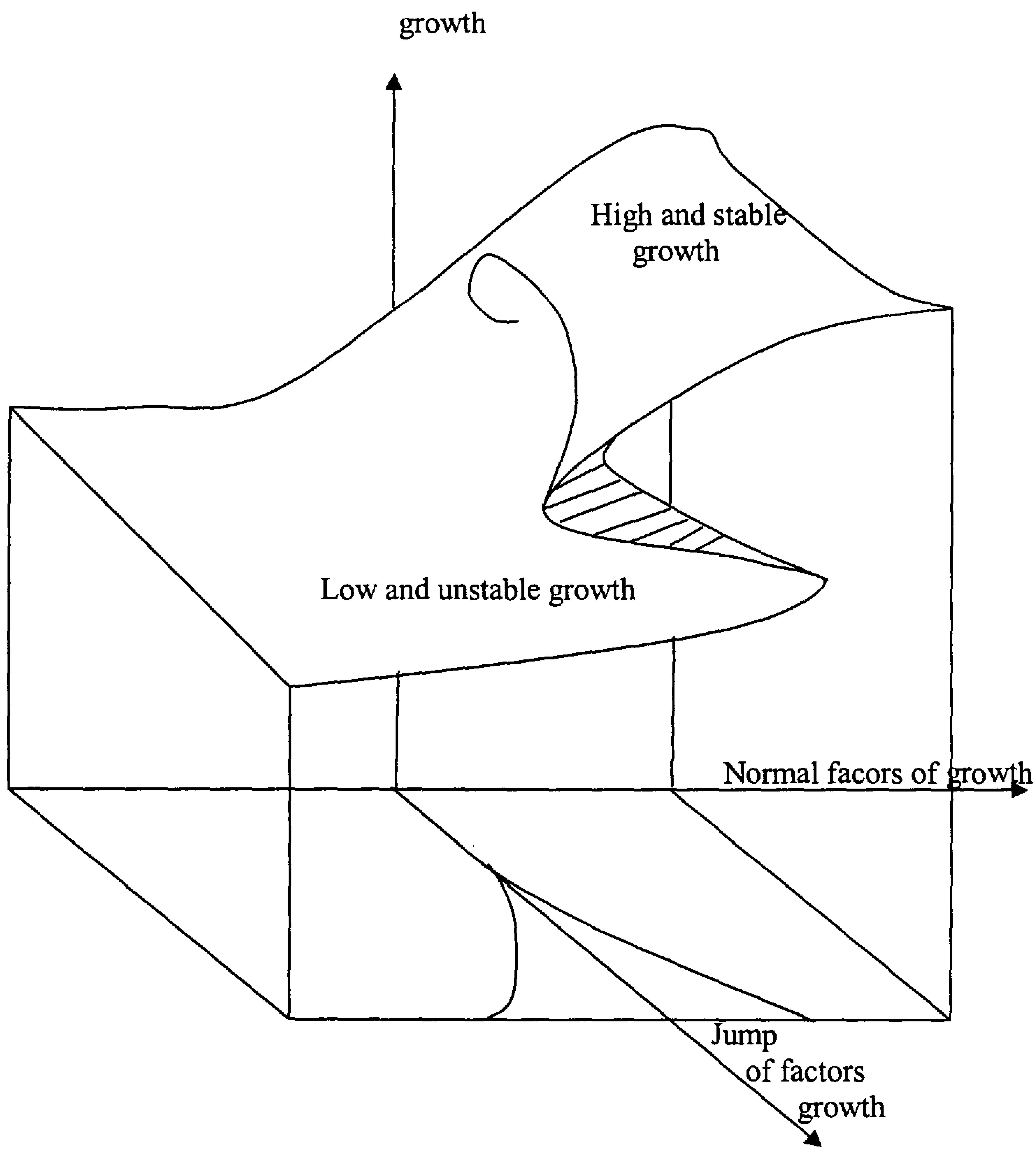
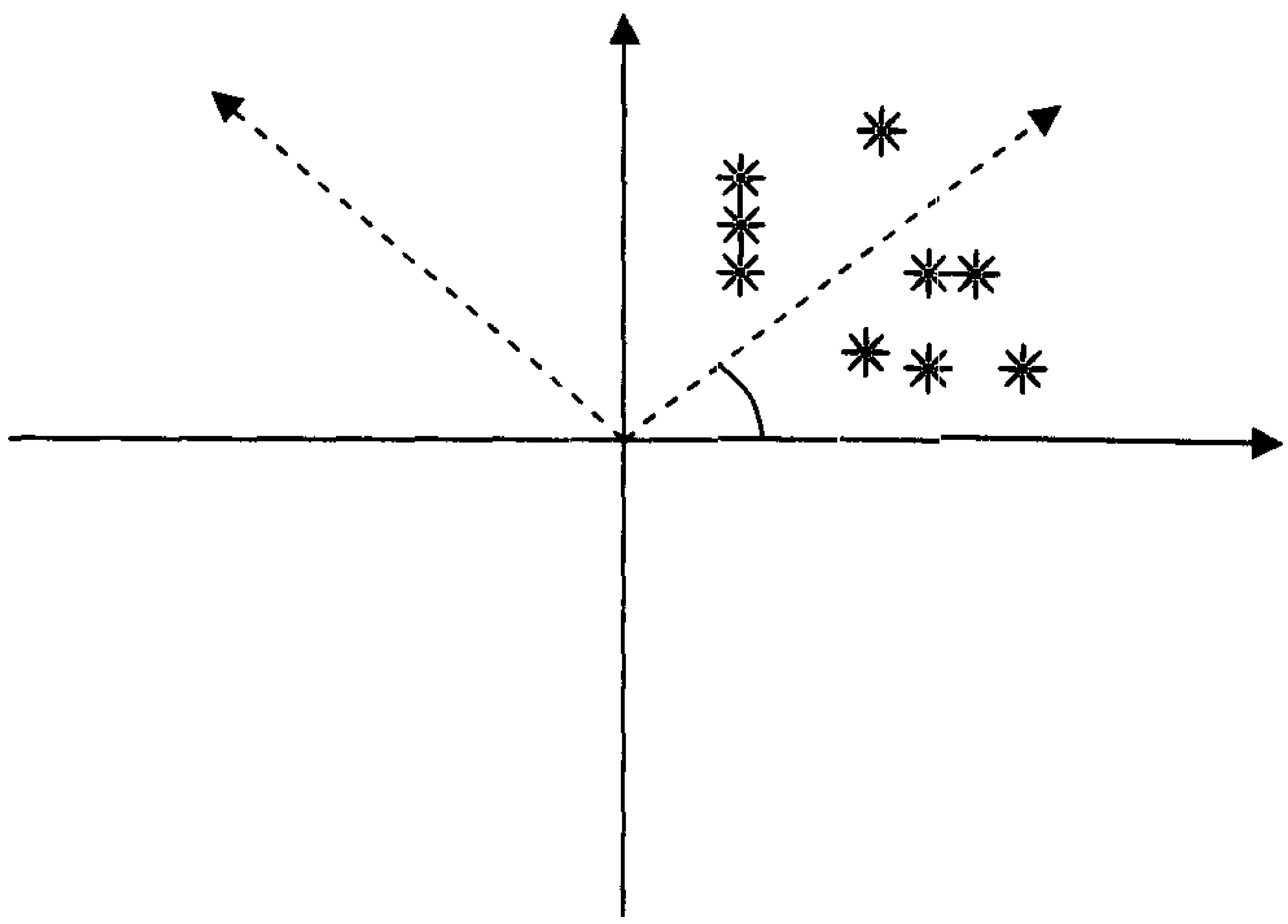
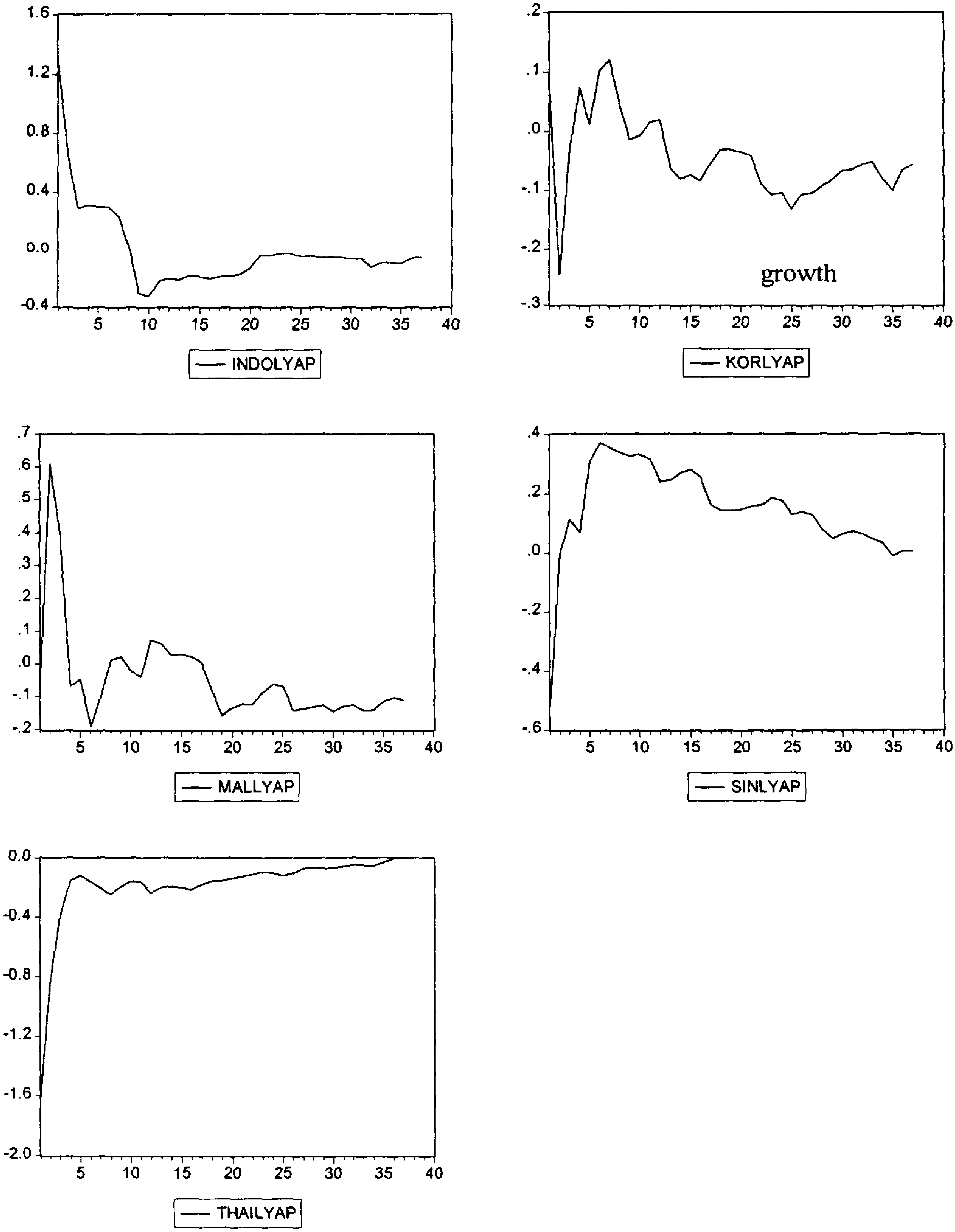


Figure 2:factor rotation and sign of control variables



In this figure before rotation all of factors are in the positive region, but after it some of them are in positive and the others in the negative quadrant.

Figur3:lyapunov exponent for GDP growth of NICs countries



References

1- Abramowitz (1986), Catching up, Forging Ahead, and Falling Behind; Journal of Economic History, 46 (June): 385 – 406.
2- Adelman, I. & Morris, C.T. (1997). Development History and its Implications for Development Theory. World Development, vol. 25, No.6, pp. 831 - 840.
3- Alexander, R.A, Glenn R. Hebert and Richard P. Deshon. An Examination of Least-Squares Regression Modeling of Catastrophe Theory, Psychological Bulletin, 1992, Vol. 111, No.2, 236-374.
4- Balasko, Y. (1978). Economic Equilibrium and Catastrophe Theory: An Introduction, Econometrica, vol. 46, No.3 (May 1978).
5- Chenery, H.B. and Syrquin, M. (1975) Patterns of Development 1950-1970 Oxford University Press.
6- Cobb, L. (1978), Stochastic Catastrophe Models and Multimodal Distribution, Behavioral Science, 23, 360 – 374.
7- Cobb, L. (1998). An Introduction to Cusp Surface Analysis, Atheling consultants, 200 oak Run Carbondale, co81623.
8- Fischer, Edwin O. and Werner jammernegg (1986), Empirical Investigation of a catastrophe theory extension of the Phillips curve, Review of economics and statistics 68, 9-17.
9- Groesen E.V. & E.M. Dejare (Eds). Studies in Mathematical Physics. Vol. 2, North - Holland.
10- Guastello, S.J. (1995). Chaos, Catastrophe and Human Affairs, Lawrence Erlboum Associates, Publishers.
11- Hirschman, A.O, and the Strategy of Economic Development (New Haven: Yale University Press).
12- Lee, T.H., H. White, and C.W.J. Granger, Testing for Neglected Nonlinearity in Time Series Models. Journal of Econometrics, 56(1993) 269-290. North Holland.
13- Lundbergh, S., Timo Trasvirta. Time-Varying Smooth Transition Autoregressive Models. Journal of Business & Economic Statistics. January 2003, Vol.21, No.1, pp.104-121.
14- Mardia, K.V., J. T. Kent and J.M. Bibby (1979). Multivariate Analysis, Academic Press.
15- Poston, T. & Stewart, I., N. (1976), Taylor Expansion and Catastrophes. Pitman Publishing London.
16- Puu, T (1991). Nonlinear Economic Dynamics, Springer Verlage.
17- Rosser, J.B. Jr., M.V. Rosser, S.J. Guastello, R. W. Bond (2001), chaotic Hysteresis and Systemic Economic Transformation: Soviet Investment

Patterns. Nonlinear Dynamics Psychology, and Life Sciences, Oct. 2001, Vol. 5. No4, pp. 345-368.

18- Rosser, J.Barkeley, jr. (1991), from catastrophe to chaos: A General Theory of Economic Discontinuities, Boston: Kluwer.

19- Rostow, WW. (1960), the Stages of Economic Growth: A Non - Communist Manifesto (Cambridge: Cambridge University Press).

20- Rummel, R.J. (2000). A Catastrophe Theory Model of the Conflict Helix with Tests, Working Paper.

21- Terasvirta, T. Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models. Journal of the American Statistical Association, March 1994, Vol. 89, No.425 pp.208-218.

22- Terasvirta, T.(1998). Modeling of Economic Relationships with Smooth Transition Regressions, Handbook of Applied Economic Statistics, Aman Ullah and David E. A.Giles.

23- Terasvirta, T., Chein -Fu Lin and Clive W.J. Granger. Power of the Neural Network Linearity Test, Journal of Time Series Analysis, (1993) Vol. 14, 209-220.

24- Varian, H.R (1981). Handbook of Mathematical Economics: Dynamical Systems with Application to Economics, North - Holland.

25- Williamson, Oliver, E. (2000). "The New Institutional Economic: Taking Stock, Looking Ahead" Vol.XXXVIII .No.3.

26- Zeeman, E.C. (1974). On the Unstable Behavior of Stock Exchanges. Journal of Mathematical Economics 1 (1974): 39-46.

27- Zeeman, E.C. (1976). Catastrophe Theory Scientific American. 234, PP.65-83.