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Cartelized Oil Market with Alternative Energy Supply

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This paper presents an oil price cartel model. The aggregate reaction functions for non-cartel producers and for substitute suppliers are included. The former group acts as a price-taker, while the latter expects oil prices in production of its non-oil energy resources. This expectation about prices affects a cartel's oil demand and, thus, gives intertemporal price elasticities. It turns out that if these elasticities are positive, Hotelling's rule does not apply to a cartelized market in which a cartel behaves as a price-maker.

1. Introduction

The proposition that the net price in a competitive market and the net marginal revenue in a monopolistic market rise at the rate of interest was first demonstrated by Hotelling (1931) for a non-durable exhaustible resource.[1] This Hotelling rule applies also to a perfectly durable resource when the resource is produced in a competitive market, but does not apply to the case of a monopolistic market.

These results with constant unit extraction cost were found by Stewart (1980). His model equated the net marginal revenue with the

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1. Net price (net marginal revenue) is price (marginal revenue) less marginal cost.

discounted next-period profit margin in a monopolistic market. The same results with rising marginal and average costs were explored by Levhari and Pindyck (1981), who analyzed Hotelling's rule in detail for a competitive market regarding the extent of the durability of the resources and the characteristics of their demand curves. Moussavian and Samuelson (1984) demonstrated that this r-percent path also fails to hold for a non-durable resource with a zero extraction cost.[2]

This paper shows that Hotelling's r-percent growth rule fails to hold in a cartelized market when oil demand facing the cartel is a function of past and present prices. If intertemporal price elasticities of the cartel oil demand are zero, the r-percent rule holds under constant unit extraction cost. However, it does not hold when unit cost is rising with cumulative production.

In the next section a model is presented and the r-percent rule is analyzed with a constant unit cost. Section III demonstrates the model with extraction costs per unit of output as a function of cumulative production. Section IV offers some concluding remarks.

2. The Model and the Analysis of the r-Percent Rule

Assume oil, an exhaustible resource, is produced in part by a group of producers who have formed an oil cartel. Assume also that this group acts as a price-maker and takes the reaction functions of the non-cartel and non-oil energy producers into consideration when its members

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2. Their paper concludes that with constant demand elasticities, a non-durable exhaustible resource monopolist expects the price to grow faster (slower) than the rate of interest as the initial stock of capital is large (small).

behave as the residual demand producers.[3] Residual demand is defined in aggregate form as:

$$D_t = W_t - \bar{S}_t - S_t \quad \text{for } t=1,...,T, \qquad (1)$$

Where D_t is the residual demand at time t faced by a cartel, W_t is the total energy demand at time t, $\bar{S}_t$ is the supply function of outside cartel producers at time t, and S_t is the supply function of substitute producers in period t. Total energy demand is assumed to be a decreasing function of the energy price, which in turn is directly related to the price of oil, and, thus, energy demand becomes a decreasing function of oil prices:

$$W_t = W(P_t), \quad \text{for } t=1,...,T, \qquad (2)$$

where P_t is the price of oil in period t. The non-cartel producers act as price-takers and their aggregate supply function which is increasing in oil price is

$$\bar{S}_t = \bar{S}(P_t) \quad \text{for } t=1,...,T. \qquad (3)$$

Finally, non-oil energy (substitute) producers are assumed to produce according to their expectations about oil prices set by the cartel. By assuming that the expected price is a function of past and present prices,

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3. The residual demand relationship is common in most empirical optimization models as well as dynamic and static simulation models. For example, see Hnyilicza and Pindyck (1976), Cremer and Weitzman (1976), Eckbo (1976), Marshalla (1978), Daly, Griffin and Steele (1982), Ben-Shahar (1976), Blitzer, Meeraus and Stoutjesdijk (1975).

i.e., $P_t^* = P^* (P_1, P_2, ..., P_t)$, their aggregate supply function becomes

$$S_t = S (P_1, P_2, ..., P_t) \quad \text{for } t=1,...,T. \qquad (4)$$

Substituting (2), (3), and (4) into (1), the residual demand as a function of past and present prices is given by:[4]

$$D_t = D (P_1, P_2, ..., P_t) \qquad (5)$$

Considering 5 as a constraint of the model and letting V represent the discounted future profits during the next T periods, the cartel's problem is written as

$$\max_{\{P_t\}} V = \sum_{t=1}^{T} D_t (P_t - c) \text{D}^t, \quad \text{where } \text{D} = \frac{1}{1 + r}$$

subject to

$$\sum_{t=1}^{T} D_t \leq R_1$$

where R_1 is the given proven reserves at the beginning of the first period, c is the constant extraction costs per unit of output over the planning horizon, D and r are the given discount factor and rate. Denoting the multiplier assigned D to the depletion constraint by λ, the

4. Stewart (1980) defined the inverse demand function which is related to past and present production and specified it for a perfect durable resource as $P_t = f (q_1 + q_2 + ... + q_t)$ where P_t is the price of the resource and q_t is the extraction rate in period t.

lagrangian for solving the cartel's problem is given by:

$$L = \sum_{t=1}^{T} D_t (P_t - c)\,\textsc{d}^t + \lambda \left(R_1 - \sum_{t=1}^{T} D_t\right). \qquad (6)$$

Differentiating 6 with respect to price, the necessary conditions for profit maximization are obtained and appear as follows.

$$\frac{\partial L}{\partial P_1} = \frac{\partial D_1}{\partial P_1} (P_1 - c)\,\textsc{d}^1 + \sum_{t=2}^{T} \frac{\partial D_t}{\partial P_1} (P_t - c)\,\textsc{d}^t + \lambda \left(\sum_{t=1}^{T} \frac{\partial D_t}{\partial P_1}\right) + \textsc{d}^1 D_1 = 0$$

(6.a)

$$\frac{\partial L}{\partial P_2} = \frac{\partial D_2}{\partial P_2} (P_2 - c)\,\textsc{d}^2 + \sum_{t=3}^{T} \frac{\partial D_t}{\partial P_2} (P_t - c)\,\textsc{d}^t - \lambda \left(\sum_{t=2}^{T} \frac{\partial D_t}{\partial P_2}\right). + \textsc{d}^2 D_2 = 0$$

(6.b)

$$\frac{\partial L}{\partial P_T} = \frac{\partial D_T}{\partial P_T} (P_T - c)\,\textsc{d}^T + D_T \textsc{d}^T - \lambda \left(\frac{\partial D_T}{\partial P_T}\right) = 0$$

(6.c)

Eliminating λ between (6.)a and (6.)b and denoting the marginal revenue by:

$$MR_i = P_i + \left(\frac{\partial P_i}{\partial D_i}\right).D_i$$

and the oil revenues by I_i $(i=1,2)$, profit maximization requires that

$$MR_2 - c = \textsc{d}^{-1} (MR_1 - c) + F_1 \qquad (7)$$

where

$$F_1 = \frac{\mathrm{D}^{-1} I_1 \sum_{t=3}^{T} D_t e_{t2} - I_2 \sum_{t=2}^{T} D_t e_{t1} - \mathrm{D}^{-1} D_1 e_{11} \sum_{t=3}^{T} D_t e_{t2} h_t}{D_1 e_{11} . D_2 e_{22}}$$

and $$e_{t1} = \frac{\partial D_t}{\partial P_1} \cdot \frac{P_1}{D_t} \qquad \text{for } t=2,3,\ldots,T$$

and $$e_{t2} = \frac{\partial D_t}{\partial P_2} \cdot \frac{P_2}{D_t} \qquad \text{for } t=3,4,\ldots,T$$

are the intertemporal price elasticities. If these elasticities are assumed to be fixed over time, i.e. $e = e_{t1} = e_{t2}$, then a simpler form for F_1 will be given by:

$$F_1 = e \left(\frac{\mathrm{D}^{-1} I_1 R_3 - I_2 R_2 - \mathrm{D}^{-1} D_1 e_{11} \sum_{t=3}^{T} D_t h_t}{D_1 e_{11} . D_2 e_{22}} \right) \qquad (8)$$

where R_i is the available proven reserves at the beginning of the ith period (i=2,3), $e_{ii} = - \frac{\partial D_i}{\partial P_i} \cdot \frac{P_i}{D_i}$ is the price elasticity of demand at time i (i=1,2), and $h_t = (P_1 - c) - \mathrm{D}^{t-1} (P_t - c)$.

It follows from (7) that in a cartelized market for oil resources, Hotelling's r-percent growth rule does not hold as the intertemporal price elasticities are different from zero. These elasticities are derived from a residual demand which is a function of past and present prices.

These past and present prices that affect such a demand result from the oil price expectations of the substitute producers. Under these price expectations, the cartel cannot apply Hotelling's r-percent rule to the world oil market. But without any reactions from alternative energy producers, this rule becomes:

$$MR_2 - c = \text{D}^{-1}\,(MR_1 - c) \qquad \text{iff } e = 0.$$

This shows that the net marginal revenue is seen to grow at a rate equal to the market interest rate when constant intertemporal price elasticities are zero.[5] In this case Hotelling's r-percent path holds and an oil cartel acts as a perfect monopoly.

3. Model with Rising Unit Cost

Consider again the model with extraction costs per unit of output as a function of cumulative production. Let unit extraction cost in period t be $C_t = C\,(Q_t)$ where $Q_t = \sum_{i=1}^{t} D_i$ is the cumulative production in

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5 . In (8) if the expression inside the parenthesis is set to zero, then

$$e_{11} = \frac{I_1 R_3 - \text{D} I_2 R_2}{D_1 \sum_{t=3}^{T} D_t h_t}$$

and marginal profit will rise over time as fast as the market interest rate.

period t. It is assumed that C is an increasing function of Q_t, i.e. $\frac{\partial C_t}{\partial Q_t} > 0$ which implies that unit costs rise due to an additional increase in cumulative production. Under this assumption and defining $f_t = \delta^t D_t \frac{\partial C(Q_t)}{\partial Q_t}$ for t=1,2,...,T, the profit maximization strategy requires that:

$$MR_2 - MC_2 = \delta^{-1} (MR_1 - MC_1) + F_1 + F_2$$

where F_1 is shown in (7) and the form of F_2 is written as

$$F_2 = \frac{-f_2 D_2 e_{22} + f_1 \sum_{t=3}^{T} D_t e_{t2} + \sum_{j=2}^{T-1} f_j \sum_{i=j+1}^{T} D_i e_{i2}}{\delta^2 D_2 e_{22}}$$

As in the previous section, by using the assumption of constant intertemporal price elasticities of the cartel oil demand curve, we will obtain an alternative expression

$$F_2 = \frac{-f_2 D_2 e_{22} + e \left[f_1 R_3 + \sum_{j=2}^{T-1} f_j R_{j+1} \right]}{\delta^2 D_2 e_{22}}$$

where $R_{j+1} = \sum_{i=j+1}^{T} D_i$ is the available proven reserves in the beginning of the (j+1)th period. With zero intertemporal price elasticities, net marginal revenue will not rise over time at a rate equal to the market interest rate:

$$MR_2 - MC_2 = D^{-1}(MR_1 - MC_1) - \frac{f_2}{D^2}$$

Since unit costs are rising with cumulative production, $\frac{f_2}{D^2} > 0$ and net marginal revenue is seen to rise at a slower rate than the market rate of interest.

4. Conclusion

In the world energy system, an oil cartel is a residual demand producer. Its net marginal revenue is not rising at the rate of interest when demand for its oil depends on expectations of alternative energy producers about oil prices. With zero intertemporal price elasticities of the cartel oil demand, the cartel's marginal profit grows over time at a rate equal to the market interest rate when its unit cost is constant. But this proposition does not hold by assuming zero elasticities and rising unit costs with cumulative production.

In an application of this model to the world energy market, OPEC is the world's residual source of oil. World demand for OPEC's oil is the difference between world energy demand and the supplies of non-OPEC oil-producing countries as well as supplies of substitute producers. Substitute producers include most developed nations with higher needs for oil consumption. These countries first prefer to purchase oil from non-OPEC countries (like Mexico, Alaska, Norway, and the North Sea) or to use their own substitute products (including natural gas, coal, hydroelectricity, and nuclear power) for offsetting additional oil consumption. If these sources are not sufficient to satisfy their additional oil demand, they purchase oil from OPEC nations. One of the reasons for switching into the other sources of energy or to sources outside of OPEC is the expectations of higher OPEC oil prices. In the current oil market this reason may not be as relevant because of declining OPEC oil prices, but it is quite possible that in the future substitutes for OPEC oil may again be a valid concern.

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