Investigating Chaos in Tehran Stock Exchange Index

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Abstract

Modeling and analysis of future prices has been hot topic for economic analysts in recent years. Traditionally, the complex movements in the prices are usually taken as random or stochastic process. However, they may be produced by a deterministic nonlinear process. Accuracy and efficiency of economic models in the short period forecasting is strategic and crucial for business world. Nonlinear models are efficient enough and suitable for short time forecasting. So notable attempts is devoted on understanding different economic time series' and nonlinear dynamical models that can fit them.

In this paper, it is tried to investigate Tehran stock exchange index time series. It is assumed. So, the Correlation Dimension (CD), the Hurst Exponent, and the Largest Lyapunov Exponent (LLE) of the time series are calculated. It is shown that the time series corresponding to Tehran stock Exchange index is nonlinear. The analyses of the results show enough evidence to accept the conjecture of existence chaotic behavior in Tehran stock exchange index.

Keywords: Chaos, Largest Lyapunov Exponent, Hurst Exponent, Correlation Dimension, Time series, Tehran stock Exchange Index.

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1 - Introduction

Recently many researchers in different fields of behavioral science and engineering use nonlinear systems theory for modeling and forecasting. Rich behaviors of nonlinear systems are the main reason for this attempt. Using this theory in economics is one of the tough and hardest one. Because one is encountered by lack of long-enough data series, huge amount of noise in data sets, lack of strong statistical tests to approve nonlinear phenomena such chaos in short data series. However, some researchers such as have tried to produce long financial data by testing in artificial financial market (Chen (2001)).

Pervious works on applications of nonlinear theory in business & financial world can be categorized in time series' investigation, improving models. Here, we use this theory for time series investigation.

Since the publications of Frank and Stengos (1989) paper, who found evidence of non-linearity in the Rate of Return (RoR) of silver and gold, economists are looking for nonlinear models for economic data. Many researchers, such as Scheinkman & LeBaron (1989 a,b), Blank (1991), Hsieh (1991), DeCoster (1992), Yang and Brorsen (1993), Fang (1994), Kohzadi (1995), Panas, E. & Ninni, V.(2000), successfully found strong evidence of nonlinearities in economic time series. "The effort devoted to study time series reflects the fact that nonlinearities convey information about the structure of the series under study". (Panas, E. and Ninni, V., 2000).

There are several tests for investigating nonlinear behaviors of time series. We use the following tests to investigate Tehran stock exchange index: Hurst Exponent, Correlation Dimension (CD) and Largest Lyapunov Exponent (LLE).

Hurst exponent can distinguish between stochastic and deterministic data. Correlation Dimension can find embedding dimension and indicate as a measure for the strangeness of an attracter. Largest Lyapunov Exponent is suitable for testing of convergence or divergence of trajectories in phase space, and calculating the information memory.

Our main data is daily Rate of Return (RoR) on main index. However, to have better understanding of time series behavior we also analyze the below data Tehran stock exchange main index, Daily RoR of main index, weekly RoR of main index, Monthly RoR of main index, Shuffle data of index, Residue of main index RoR.

To compute LLE, we use Wolf algorithm (1985). For calculating, CD, we use Grassberger and Procaccia method (1983). In Sections 2 to 4 Hurst exponent CD, and (LLE) are discussed respectively. In Section 5, the results of analysis on Tehran stock exchange index are presented.

Since most of the above computations are based on computer, developing some toolkits to facilitate the related cumbersome calculations is useful. Users may change desired variables and monitor their effects quickly. These toolkits, which offer many graphical facilities, gives the user the capability to monitor and control the key parameters of the time series such as the sample number. Hence, a toolkit in Matlab is developed to facilite the calculations.

2 – The Hurst Exponent

H.E. Hurst is responsible for a measure of predictability of time series that has interesting characteristics. The exponent is derived using so-called R/S analysis. Given a time series X containing a number of points, n, and choosing an integer divisor p where for convenience: $10 <= p < n/2$, the data can be divided into n/p blocks. For each block the average value is calculated, then the maximum range of each block and the standard deviation of each block. The value (range)/(standard deviation) is calculated for each block and then averaged over the blocks. This average value R/S is related to the Hurst exponent by the following formula:

$$R/S = \left(\frac{p}{2}\right)^{H}$$

Where H is the Hurst exponent. In order to gain a more reliable estimate the value of R/S is calculated for all the possible values of p, and the resulting tuples and logged and a linear regression is performed on them. It is the gradient of the regression line that is used as the Hurst exponent. Hurst exponent values range between 0 and 1. A value of 0.5 indicates a true random walk, a value $0.5 < H < 1$ indicates so called persistent behavior, in the one can expect with increasing certainty as the value moves towards one the whatever direction of change has been current will continue. A straight line with non zero gradient would have a Hurst exponent of 1. Similarly, values $0 < H < 0.5$ indicates anti-persistent behavior, in that one can expect that whatever direction of change is current is unlikely to continue. At the limit of 0 the time series must change direction every sample.

3 – The Correlation Dimension (CD)

There are many ways to define a dimension for an attractor. It depends on average weight of the regions of the attractor. Correlation Dimension (CD), which is very easy to calculate, gives more reliable results than the other dimensions. CD can indicate as a measure for the strangeness of the attractor, because it includes information about formation as well as final appearance of the attractor. (Sprott, 2003)

Correlation dimension can be calculated using the distances between each pair of points in the set of N number of points, (Grassberger and Procaccia 1983)

$$s(i, j) = |X_i X_j|.$$

A correlation function, C(r), is then calculated using,

$$C(r) = \frac{1}{N^2} \times (\text{number of pair } s(i, j) \text{ with } s(i, j) < r).$$

C(r) has been found to follow a power law:

$$C(r) = k\, r^D$$

Therefore, we can find D_{corr} with estimation techniques derived from the formula:

$$D_{corr} = \lim_{r \to 0} \frac{\log(C(r))}{\log(r)}.$$

C(r) can be written in a more mathematical form as

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{j=1}^{N} \sum_{i=j+1}^{N} \theta(r - |X_i - X_j|),$$

where θ is the Heaviside step function described as,

$$\theta(r - |X_i - X_j|) = \begin{cases} 1 & 0 \le (r - |X_i - X_j|) \\ 0 & 0 > (r - |X_i - X_j|) \end{cases}.$$

To get sure of chaotic structure it is better to apply CD on residue value of Auto Regression model (Brock W., Sayers C, 1998) and on shuffle values (Scheinkman j., Le Baron B, 1989). If CD of residues mature on point and it is stable, it is a good evidence for our pervious job. If CD of shuffle values is more than CD of normal data, it is evidence that the pervious results are ok.

An important point in financial time series applications is the number of data points required for good estimation of the dimension. The question of the length of the scalar time series has been a major topic of debate (Wolf A. 1985) and Ruelle (1981) suggest a minimal data length of $n \ge 10^{CD}$.

4 – The Largest Lyapunov Exponent – (LLE)

The LLE have additional features for detecting deterministic behavior. These exponents measure the exponential divergence and convergence of nearby orbits. LLE may be calculated in two cases: First when there is a mathematical differential equations for the system, and second case when an experimental data estimation of the system exists.

For the second case Alan Wolf and his colleges (Wolf, A., Swift, J.B., Swineey, H.L. and Vastano, J.A. 1985) present an algorithms to estimate the positive Lyapunov exponents from an experimental data set or a time series. Their method monitors the long-term growth rate of small volume elements in an attractor. Here, the wolf is method is used to estimate LLE.

First, we make a [m*(N-m+1)] dimension matrix from N scalar data, then we find pairs vectors that satisfy:

$$r_0(m; i, j) = |x_i - x_j| < E$$

E is a small positive value. By the growth of time length we can calculate $r_n(m;i,j)$ using the below equation:

$$r_n(m;i,j) = \left| x_{i+n} - x_{j+n} \right|$$

Then we calculate the divergence of neighbor points as:

$$d_n(m;i,j) = \frac{r_n}{r_0} = \frac{\left| x_{i+n} - x_{j+n} \right|}{\left| x_i - x_j \right|}$$

If $d_n>1$ it means by increasing the time length n, the neighbor points of m dimension phase space will diverge Having dn the equation for calculating LLE is:

$$\mathrm{LE(m,n)} = \lim_{N\to\infty} \frac{1}{N-n} \sum \log d_n(m;i,j,)$$

Positive LLE is a strong evidence of deterministic chaos. In addition, it shows that nearby points of m-dimension phase space attractor will diverge. Therefore, past information will lose its effect on future after a while in m dimension. This memory period is equal to $T \le \frac{1}{LLE}$ for each dimension.

Like other tests, LLE needs enough data points and it is sensitive to noise. Wolf mentions three reasons for enough length of data series:

- a finite amount of attractor data can only define finite length scales
- the stretching and folding that is the chaotic element of a flow may occur on a scale small compared to the extent of the attractor
- Noise defines a length scale below which separations are meaningless.

5 -Data analysis

The data set consists of Tehran stock exchange index for the period 22/11/1998 to 28/02/2006. This series is obtained from Tehran stock database web site.

Before analysis of the above data, the growth of time trend and the noise should be considered. The logarithm of data is used to remove systematic calendar and trend effects. There are plenty of noise reduction and filtering techniques but we did not use them in this research.

First, we calculate RoR of daily, weekly and monthly data, which are shown in Figure 2, Figure 3, and Figure 4.

We test for stationary using the augmented Dickey-Fuller (ADF) test statistic. The ADF test statistic is -15 and it is less than the critical values, which are -3, -2, -2. It means RoR series is stationary.

The Hurst Exponent of Tehran Stock Exchange Index

The Hurst exponent is calculated for weekly and monthly RoR. The result of regression for R/S models are shown below:

Regression of Weekly RoR: $y = 0.7646x - 0.1703$ $R2 = 0.9971$

Regression of Scrambled weekly RoR: $y = 0.5751x$ $R2 = 0.9751$

Regression of Monthly RoR: $y = 0.889x - 0.5499$ $R2 = 0.9931$

Regression of Scrambled Monthly RoR: $y = 0.5885x$ $R2 = 0.971$

The results are acceptable and as we see the weekly H=0.76 is less than Monthly H=0.88

Since Hurst exponents are between 0.5 and 1, the time series is deterministic. In addition, the long memory effect calculated as:

Weekly RoR Information memory: $N^H = (1737) = 300\text{day}$

Monthly RoR Information memory: $N^H = (1717) = 751\text{day}$

It means after 300 and 751 days the current day information loses its effect on future trends.

The CD‘s of Tehran Stock Exchange Index

The correlation dimension for daily, weekly, and monthly data are shown in Figures 5 to Figure 7 respectively. It is notable that they are getting mature on zero while dimension increases. The results of correlation dimension is summarized in Table 1 to Table 3. The CD for weekly and monthly data is 2.2 and for daily RoR is 1.6. These values of CD shows indication of strangeness of the attractor associated to the time series.

The LLE of Tehran Stock Exchange Index

The result of calculation for LLE on Daily, Weekly and monthly data are shown in Table 4, to Table 6 and Figure 8, to Figure 14. LLE vs. T in each dimension is also shown. It may be found that longer RoR, smaller dimension may be achieved.

In Tables 4, 5, 6 information memory is calculated for each dimension. “T” the inverse of LEE shows how long today information can effect on future information However, keep in mind during T period the amount of information effect decrease till end of period. In Figure 21, LLE for dimension 26 of daily RoR is shown.

Based on positive evidence of CD and LLE one may conclude that the Tehran stock exchange index is chaotic and there are strong evidences of deterministic chaos.

Table 1: Daily Correlation Dimension

Dimension	5	10	15	20
CD	0.371369	0.812866	1.263043	1.690559

Table 2: Weekly Correlation Dimension

Dimension	5	10	15	16	17	18	19	20
CD	0.77203	1.3810295	1.953232	2.0287676	2.11087	2.287557	Infinity	Infinity

Table 3: Monthly Correlation Dimension

Dimension	5	10	13	14	15
CD	1.029311	1.683133	1.971164	2.172654	Infinity

Table 4: Largest Lyapunov Exponent and Information Memory for Same Dimensions of Daily RoR for Dimension 14 to 30

Dimension	14	16	18	20	22	24	26	30
LLE	0.95772	0.31171	0.085148	0.038895	0.02293	0.013579	0.008037	.0011854
T-day	1	3.2	11.7	25.7	43.61	73.6	124.4	843.5

Table 5: Largest Lyapunov Exponent and Information Memory of Weekly RoR for Dimension 5 to 15

Dimension	5	10	15
LLE	18.68069	0.413131	0.008843
T- Day		2.420542	113.0889

Table 6: Largest Lyapunov Exponent and Information Memory of Monthly RoR for Dimension 5 to 8

Dimension	5	6	7	8
LLE	0.353399	0.093252	0.027529	0.005389
T- Day	2.829664	10.72359	36.32546	185.5509

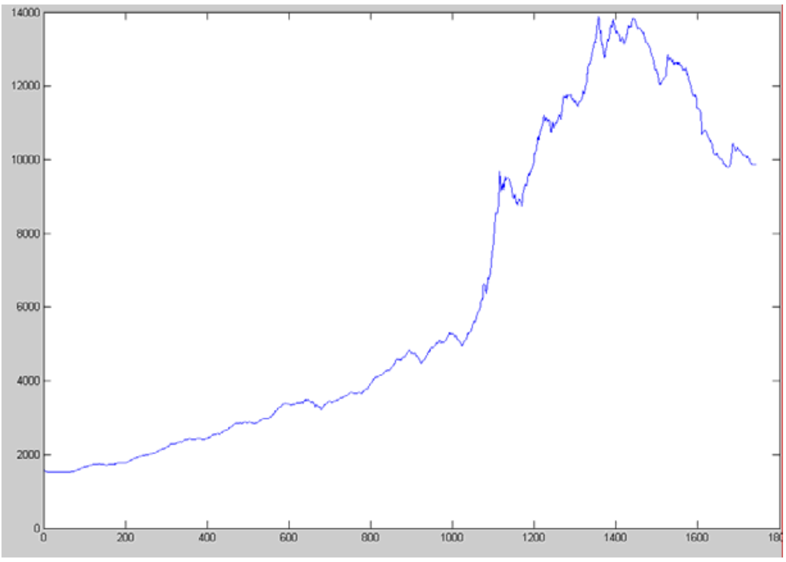

Figure 1 : Daily Index of Tehran Stock Exchange from 22/11/1998 to 28/2/2006

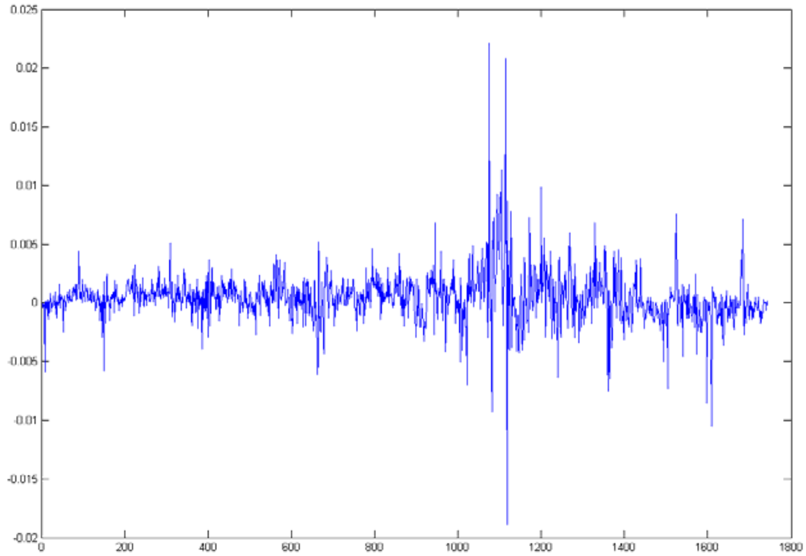

Figure 2 : Daily RoR of Main Index

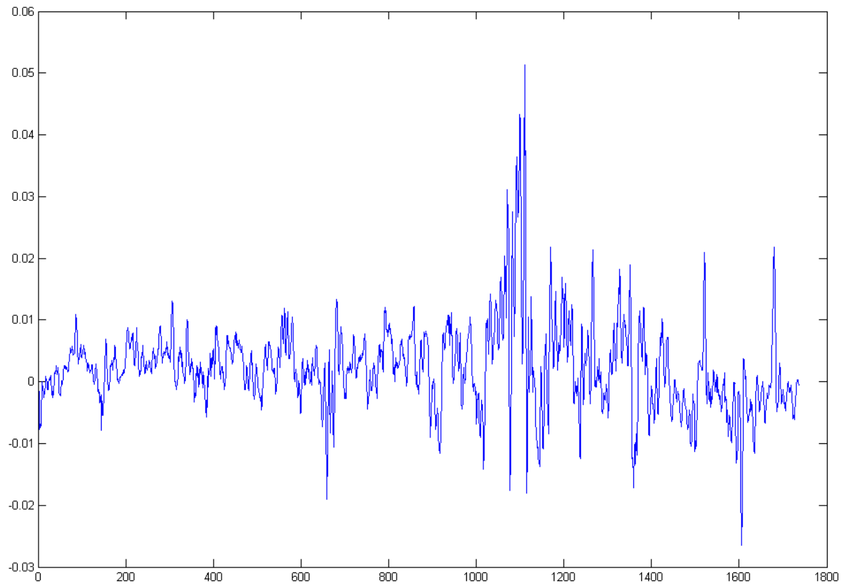

Figure 3 : Weekly RoR of Main index

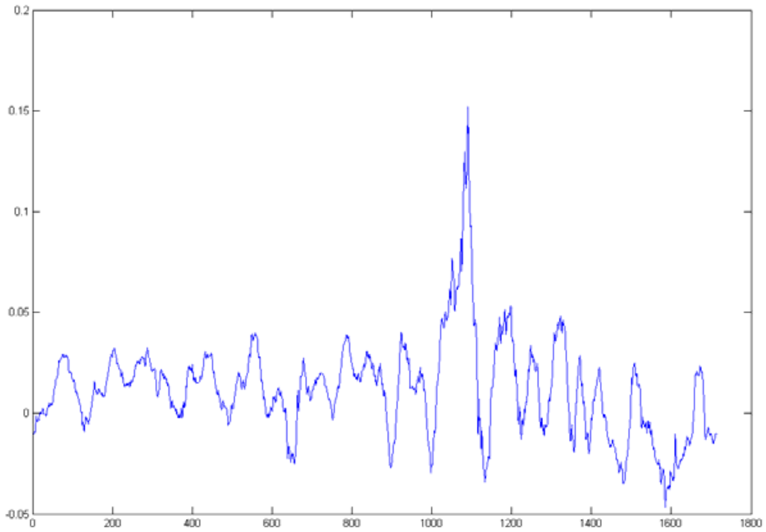

Figure 4 : Monthly RoR of Tehran index

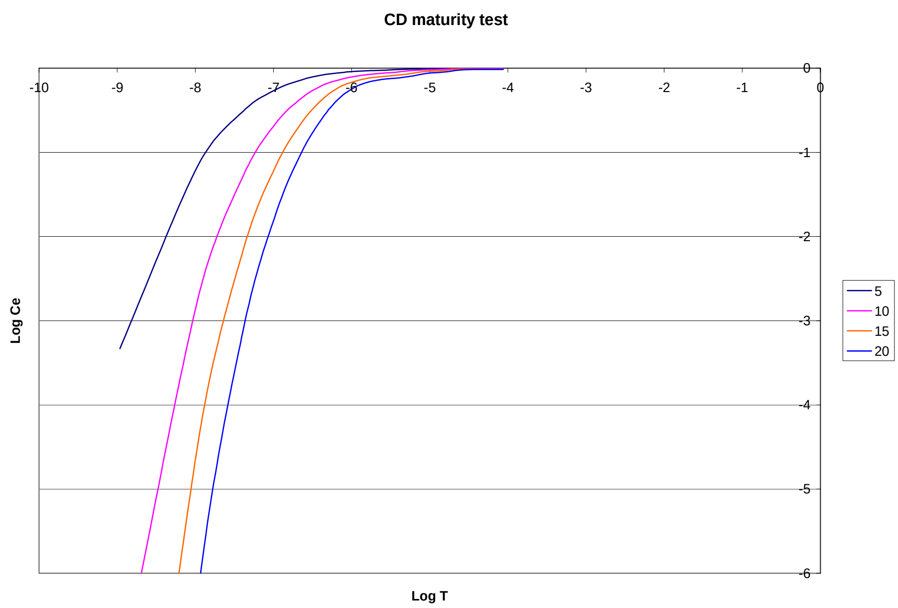

Figure 5 : Daily RoR CD Maturity Diagram

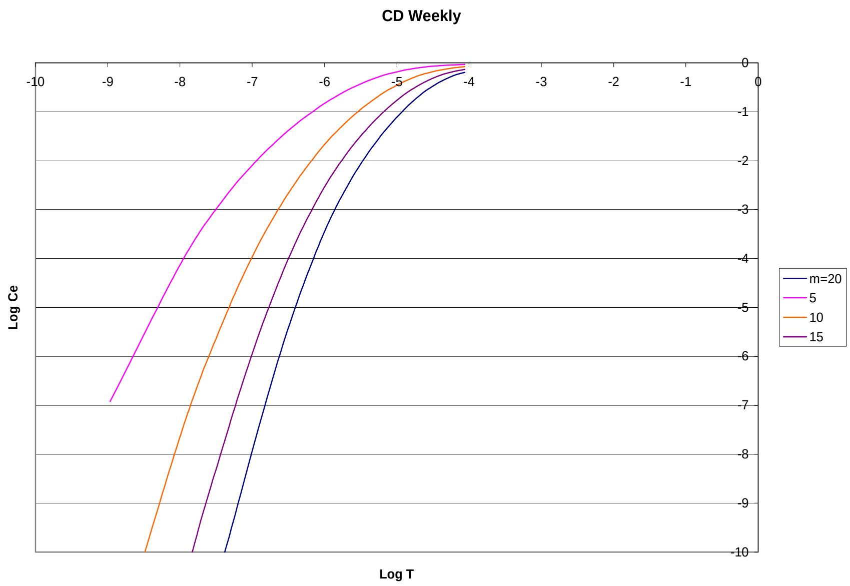

Figure 6: Weekly RoR Correlation Dimension Maturity Diagram

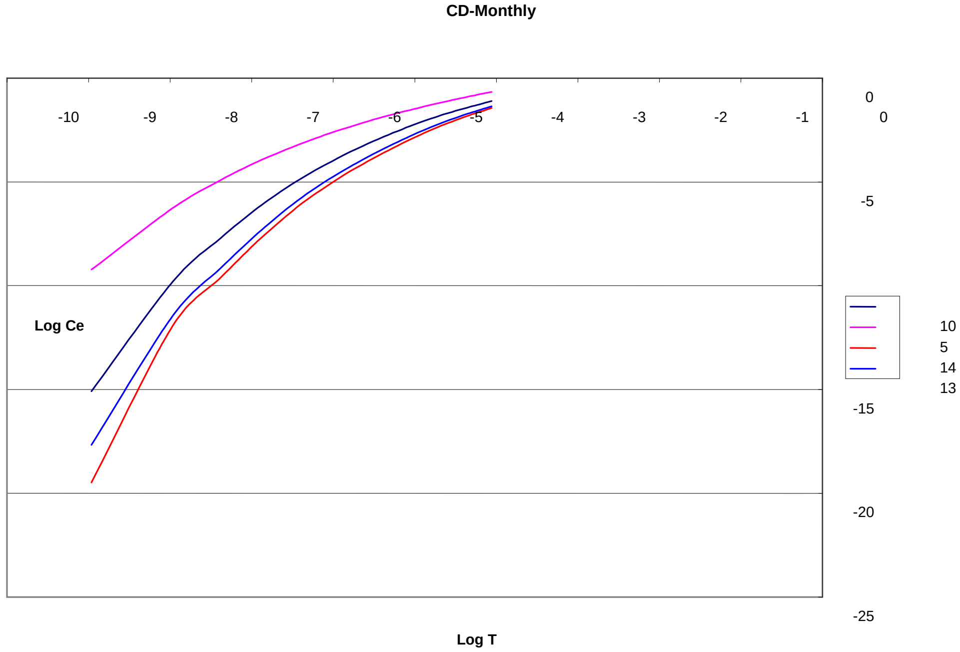

Figure 7: Monthly RoR Correlation Dimension Maturity Diagram

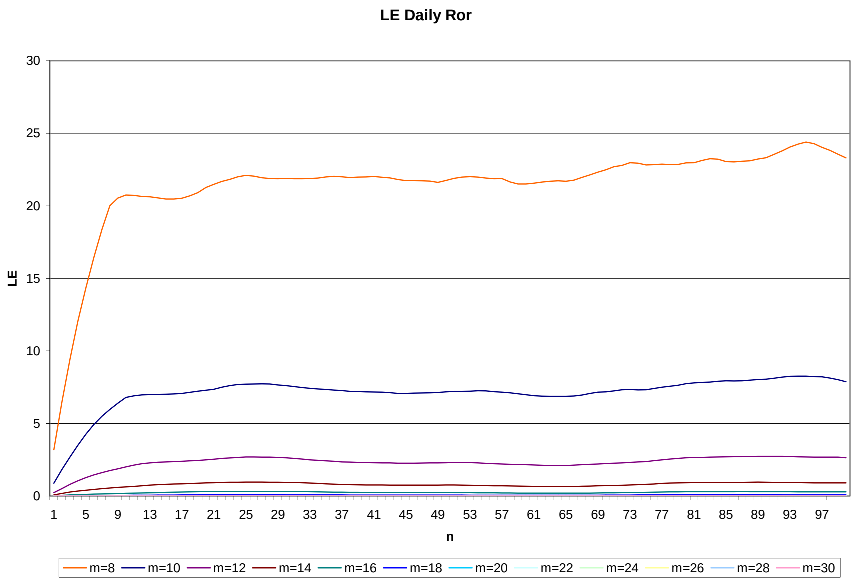

Figure 8: Lyapunov Exponent diagram for Daily RoR

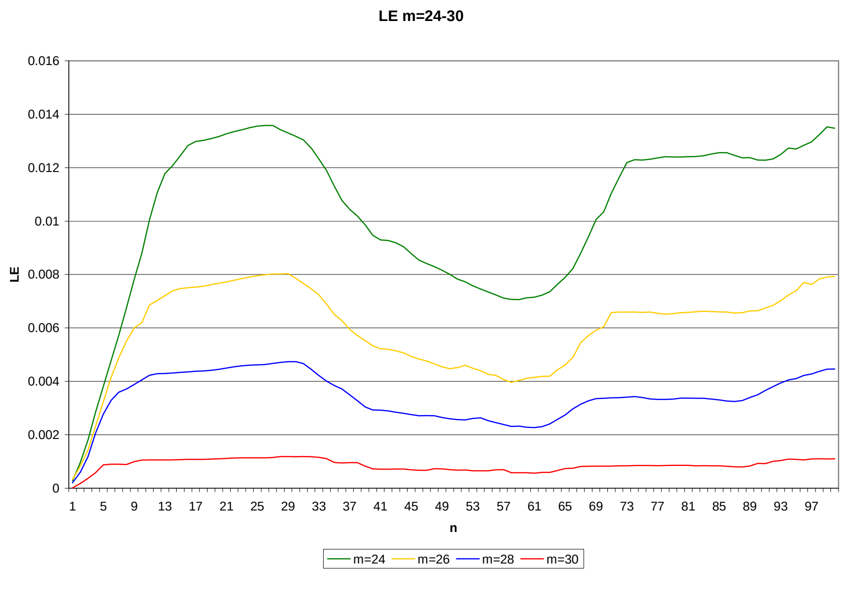

Figure 9: Lyapunov Exponent Diagram for Daily RoR -Zoom from Dimension 24-30

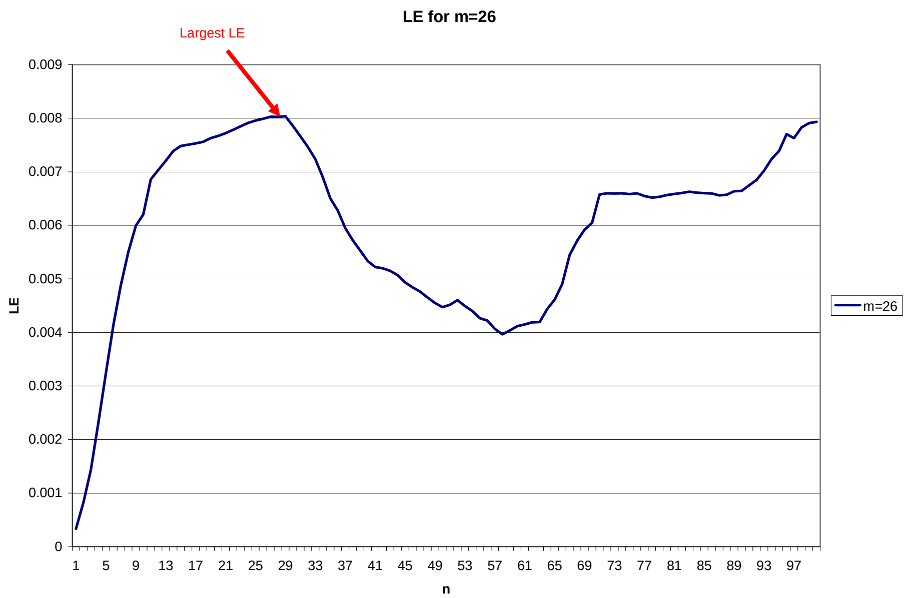

Figure 10: Lyapunov Exponent Diagram for Daily RoR - Dimension 24

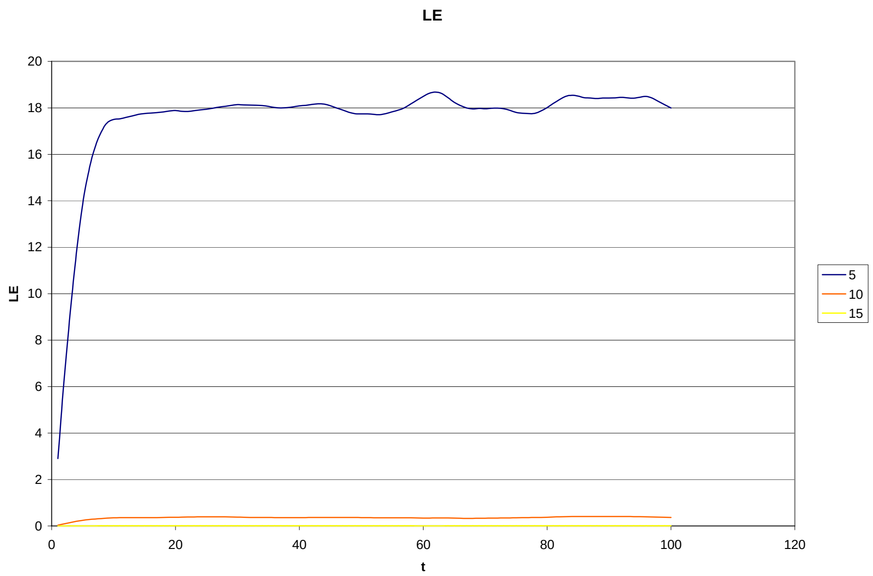

Figure 11 : Lyapunov Exponent Diagram for Weekly RoR –Dimension 5-15

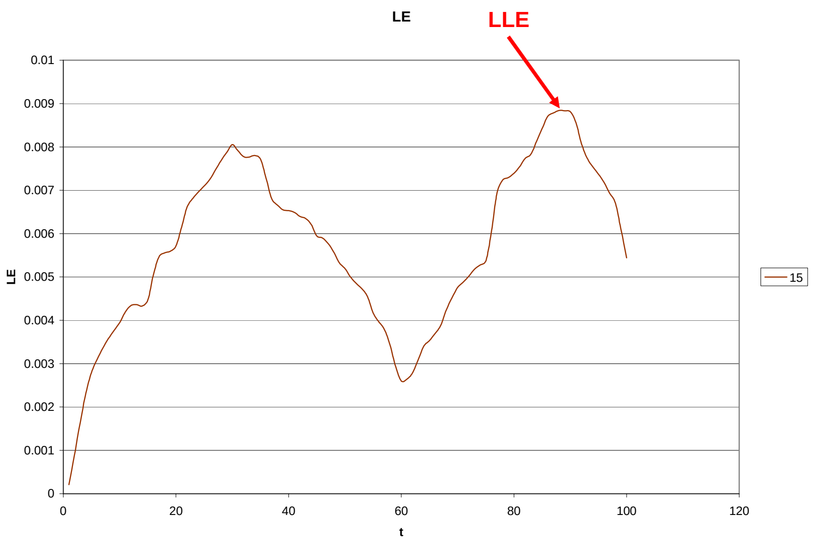

Figure 12: Lyapunov Exponent Diagram for Eeekly RoR - Dimension 15

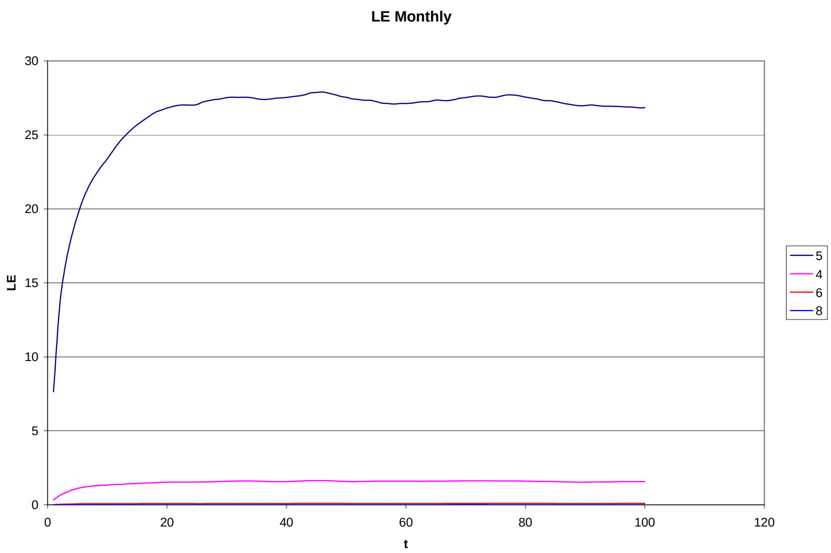

Figure 13: Lyapunov Exponent diagram for Monthly RoR –dimension 5-8

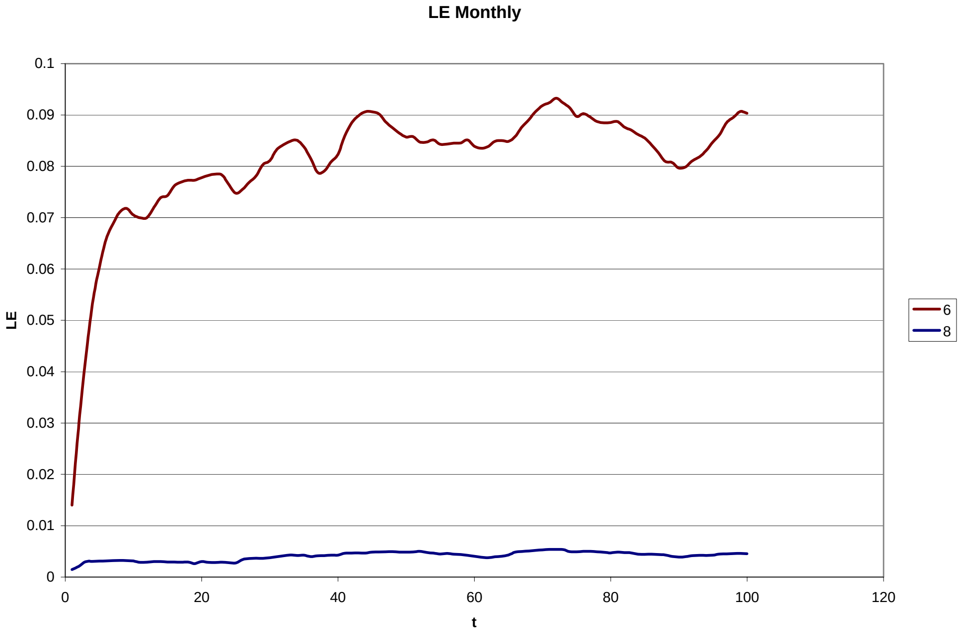

Figure 14: Lyapunov Exponent Diagram for Monthly RoR –Zoom-Dimension 6-8

Conclusion

In this paper Tehran stock exchange index is investigated by Hurst exponent, correlation dimension and largest Lyapanov exponent. By calculating the Hurst exponent and correlation dimension, it may be found that time series corresponding to Tehran stock exchange index may be considered as nonlinear deterministic series. The LLEs were calculated for different dimensions and most of them were positive. It shows an indication of chaos in time series.

It may be concluded that for forecasting in Tehran stock exchange index nonlinear and deterministic models could be more reliable, Richness of nonlinear models to interpret the behavior of time series could help researcher to understand underlying phenomena in the market. Furthermore, the assumption of determinism in the market rejects the efficient market hypothesis, prevents price manipulation, and reduces stock market inefficiency.

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