

Leading the Ignorant: Can Ignorance Eliminate the Free Riding Problem?

Mana Komai*

Abstract

This paper suggests that even if it is costless to inform all team members about the quality of a project, there are reasons to concentrate information in the hands of one person (a leader) and prevent full revelation to the rest. This deprives others of the information necessary for profitable defections; they (the followers) therefore will have no reasonable strategy other than following the informed leader because he has more information than they themselves have. Such leaders then can lead the ignorant group into cooperation in cases where information gives them an incentive not to do so. Unlike the common belief, this paper shows that lack of information transparency in a group or an organization may increase cooperation and thus efficiency compare to a regime of information dispersal.

Keywords: Leadership, Information transparency, cooperation, Incentives.

1- Introduction

Free-riding is the classic problem in groups and organizations and has been long studied in management, political science, and economics. Economists have often challenged this problem by introducing contracts, exercising formal authority, and reducing information failures. What motivates this study is that collecting and processing information is

* State University Minosota.

expensive, as are the design and enforcement of optimal contracts through monitoring and formal authority.

This paper designs a setting that exhibits the familiar problem of free-riding but provides a contrarian solution. The paper proposes a low cost environment which minimizes contracting, information exchange, monitoring, and the use of formal authority. This setting is built on leadership or informal authority. In this model rational agents choose to follow an informed leader without an obligation to do so. Leaders do not need to be special: they can just be average players who are distinguished merely by occupying the leadership position and their legitimacy can be derived, plausibly, from superior information.¹

One key feature is that, in this setting, leaders are unable to fully transmit their information to the others. This can be realistic in many contexts. In some cases the leader's information is too complicated to be fully understood by an average player. In this case the leader is practically unable to transmit his information even if he chooses to do so. In some cases the information is not complicated but difficult to verify. For example, much of the information revealed by a political figure is practically not verifiable by a potential voter. A political candidate is thus unable to credibly transmit all of his information to the voters (again even if he wants to). The same situation occurs when a person endorses a charitable foundation or a public project (a person's endorsement is a positive signal of the quality of a public project but does not reveal the project's exact rate of return).

The main point of this paper is that leaders may lead more effectively when their information is not fully transparent to their followers. In other words, ignorant agents may make better followers. The reason is that lack of information transparency can deprive the followers of the information necessary for profitable defections and may make it hard for them to protect their self interest by defecting from cooperation. They thus follow the leaders who act on behalf of the group. This may benefit the collective by

1- For example political leaders are in general more informed about a country's state of fundamentals than the public is or upper level managers are more informed about the projects in hand than their workers are.

improving cooperation even though the underlying payoff structure may potentially give every agent an incentive to shirk.¹

Many political and management scholars argue for information transparency and against politicians or managers' privileged access to information (one example is Case (1995)). This paper, however, presents a contrarian view: our paper argues that groups may benefit from ignorance, and thus it suggests that in some circumstances it is reasonable to centralize information in the hands of a person and prevent information transparency.²

A rich literature describes many potential advantages to ignorance. It is well known, at least since Hirshleifer (1971), that many situations exist in which information failures can increase efficiency; The classic example is the provision of insurance. More recent examples are: Levy and Razin (2004), Daughety and Reinganum (2006), Crawford and Sobel (1982), Austen-Smith (1994), Blume, Board, and Kawamura (2007). This paper is also related to the idea of information cascades, but unlike the many studies emphasizing the inefficiencies caused by cascades, it uses the leader-follower relationship to improve efficiency.

This is not the first attempt to understand leadership. Leadership has been long studied in management and political science but has recently

1- This is not to say that following a leader is socially optimal in all circumstances. In our homogeneous model, the leader is simply an average player with the same payoff structure of the others. Such leader well represents the public and will not start projects that are beneficial to himself but are harmful to the rest. When a population is sufficiently heterogeneous, a non representative leader may start projects which do not benefit the collective. In such cases, cooperation may be desirable to the leader but may not be socially optimal. The war in Iraq may be a good example: lack of information caused the American public to follow their leader in a project which they would have avoided if they were fully informed. The result, however, is not socially optimal for the United States since they followed the wrong leader: a leader who did not represent the benefit of the public. In heterogeneous populations, the best leader may simply be an average player who represents the incentives of an average agent.

2- Prendergast (1993) presents a quite different model with a related message. He shows that if managers rely on information provided by workers, then workers' incentive to conform means that it may be best to insulate them from managers' other sources of information.

received attention in economics.¹ Most economic research on leadership focuses on how leaders lead. A seminal work by Hermalin (1998) plausibly suggests that followers voluntarily follow a leader because they believe that the leader has better information about what they should do than they themselves have.² Hermalin focuses then on how a leader can convince his followers that he is not misleading them in situations where the leader has an incentive to do so. Hermalin considers a team leader who has private information about the return to the projects, and whose payoffs increase in his follower's effort. He suggests that the leader can induce a following by "leading by example", meaning that the leader puts in more (observable) effort when the return is high to show his followers that the project is worthwhile. He shows that the leader-follower equilibrium produces more efficient outcomes than does the equilibrium under full information, mainly because it improves the leader's incentives to work. In Hermalin's model the leader fully reveals his information (followers will not remain in dark) and welfare improves only because the leader works harder not his followers.

Vesterlund (2003) applies Hermalin's idea to a model of charitable contributions, but in her model the leader chooses whether to acquire information before deciding whether to contribute. Vesterlund focuses on whether a third player, a fundraiser who moves first, chooses to announce

1- Stackelberg (1934) is an early exception. Recent notable examples are Hermalin (1998; 2007), Wilson and Rhodes (1997), Foss (2001), Arce (2001), Rotemberg and Saloner (1993; 2000), Meidinger and Villeval (2002), Kobayashi and Suehiro (2005), Komai, Stegeman, and Hermalin (2007), Komai and Stegeman (2004), Komai, Grossman and Deters (2006), Komai and Grossman (2007), and Huck and Biel (2006). Some of the literature on sequential provision of public goods and public bads can also be seen as analyzes of strategic leadership. Some examples are Romano and Yildirim (2001), Vesterlund (2003), Moxnes and Van Der Heijden (2003), Andreoni (2006), Potters, Sefton, and Vesterlund (2005, 2006), Guth, Levati, Sutter, and Van Der Heijden (2007), and Brandts, Cooper, and Fatas (2006).

2- In some articles, leadership is not about superior information. Leaders can be simply first movers with visible actions. Some examples are Romano and Yildirim (2002), Moxnes and Van Der Heijden (2003), and Guth, Levati, Sutter, and Van Der Heijden (2003).

that the leader's contribution will be public. Andreoni (2006) builds on Vesterlund's (2003) model by endogenizing the selection of the leader.

In a theoretical setting, Komai, Stegeman, and Hermalin (2007, henceforth KSH), study a team production problem in which the leader leads by example but the leader's information is only partially revealed to his ignorant followers. The paper shows that by preventing full revelation of the state, the followers are induced to work harder than they would were the leader's action to reveal the state fully. Thus, the focus of their paper is on how to get the followers to work harder: followers will work harder if the leader leaves them partly in dark. The article sheds light on the reasons for a leader, and takes the contrarian view that concentrating information in the hands of a leader and lack of information transparency can improve efficiency versus a regime of information dispersal. The article described above (KSH) makes the restrictive assumption that players' utility is linear in actions, which implies that players' action set is binary. The current paper follows the same theme but extends the idea to a quasi-concave utility function which generates a continuous and thus a more realistic action set.¹

Komai, Grossman, and Deters (2006, henceforth KGD) design an experiment to test the theory introduced by KSH and empirically show that ignorant followers indeed follow their leaders more often.² KGD is different from the current paper for two reasons: first, unlike the current paper but as in KSH, in KGD players' action set is binary and second, unlike the current paper which is theoretical, KGD is an empirical work.

1- Komai and Stegeman (2004) introduce the problem of coordination failure and show that giving the leader private information and preventing full revelation can solve problems of moral hazard and coordination simultaneously. They also show that in a heterogeneous population the most credible leaders should not be more enthusiastic than an average player: the leaders who lead least lead best.

2- Komai and Grossman (2007) empirically show, however, that such leaders lose their effect gradually as the group size increases. In a simultaneous and independent work, Potters, Sefton, and Vesterlund also show, empirically, that leaders improve cooperation only in the presence of asymmetric information, not when information is equally available to their followers.

This paper is organized as follows. Section II presents our complete information model. Section III introduces our leader-follower setting and section IV concludes.

2- The Complete Information Model

In this section, we develop a familiar model of a group project under complete information. Consider $m+1$ identical players $I=\{0,1,2,\dots,m\}$. Each player divides his endowment w between consumption of a private good, $y_i \geq 0$, and contribution to a group project, $x_i \geq 0$. Therefore, $x_i = w - y_i$.¹ The utility function of each player has the following form: $V(x_0, x_1, \dots, x_m) = \alpha \sum x_i + U(w - x_i)$, where α is the marginal return to the aggregate contribution to the group project.

We assume that α is distributed on the interval $[0, \bar{\alpha}]$ with a continuous and strictly positive density function $f(\alpha)$. We also assume that $U: \mathbb{R}_+ \rightarrow \mathbb{R}$ is a C^3 function, $U' > 0$ and $U'' < 0$ over the interval $[0, w]$. Furthermore, we assume that $\lim_{y \rightarrow 0} U'(y) = +\infty$ (when y approaches zero), $\lim_{y \rightarrow \infty} U'(y) = 0$ (when y approaches infinity),² and players' absolute risk aversion is non increasing, implying $U''' > 0$.³ Finally, we assume that $E(\alpha) < U'(w) < \bar{\alpha}$. This assumption implies that players are willing to contribute to the project if and only if they learn sufficiently favorable information about α . The above assumptions are typical for a model representing a group project.

We consider a simultaneous move game: α is determined by nature, all players observe α , and then simultaneously decide how much to contribute to the public project.

Consider the maximization problem of a representative player. Define $\bar{\alpha} = U'(w)$. Clearly, for $\alpha > \bar{\alpha}$ each player's optimal contribution level is

1- w , x_i , and y_i can have various interpretations: w can be a player's time endowment, x_i can be his effort measured in terms of time spent on a project, and y_i can be his leisure.

2- The limit assumptions are only technical assumptions to ensure the existence of an interior solution.

3- Player i's absolute risk aversion is non-increasing if $\frac{-U'''U' + U''^2}{U'^2} \leq 0$. Since $U' > 0$ by assumption, U'' should be positive for the inequality to hold.

strictly positive and is the unique solution to the following first order condition: $\alpha = U'(w-x_i)$. For $\alpha \leq \underline{\alpha}$, players' optimal contribution is zero.

Let $X(\alpha)$ be the optimal contribution of a representative player. The unique Nash equilibrium of this game is the symmetric strategy profile $(X(\alpha))$. Note that $X(\alpha)$ is a dominant strategy; $X(\alpha)=0$ for $\alpha \leq \underline{\alpha}$ and $X(\alpha)>0$ for $\alpha > \underline{\alpha}$.

One motivation for this work is the observation that players may free ride meaning that they may refuse to make a positive contribution even when it is efficient to do so. To see this define $G(\alpha, x) = (m+1)[\alpha(m+1)x + U(w-x) - U(w)]$ to be the welfare gain obtained by the group if all players contribute $x>0$. The free-riding problem occurs because $\partial G(\alpha, x)/\partial x > 0$ implying that $G(\alpha, x) > 0$ for some $\alpha < \underline{\alpha}$ and $x > 0$, but players refuse to contribute to the project at the equilibrium when $\alpha < \underline{\alpha}$.

In the next section, we show that a leader who is given exclusive information about α can increase contributions via an initial endorsement which partially reveals his information to the others.

3- The Leader-Follower Model

This section pursues the idea that concentrating information (about α) in the hands of a leader and preventing full revelation can improve cooperation compare to a regime of information dispersal.

We consider the model from section II but revise the timing and the information structure of the game. In the new scenario, α is observed by only one player (the leader). The distribution of α is common knowledge.

In the first stage of the game, the leader makes two separate decisions: whether to endorse the project and how much to contribute. The leader's endorsement strategy is $D:[0, \alpha] \rightarrow \{0, 1\}$. The value of $D(\alpha)$ is equal to 1 if the leader endorses the project and 0 if he does not. The leader's contribution strategy is $X_0:[0, \alpha] \rightarrow \mathbb{R}_+$. The value of $X_0(\alpha)$ is 0 if the leader does not contribute to the project and is a positive number if he does. The leader's endorsement is assumed to be costly (One might think of it as the reputation that the leader loses if he endorses a low return project). The endorsement cost $R:[0, \alpha] \rightarrow \mathbb{R}_+$ is specified as: $R(\alpha) = m\theta D(\alpha)r(\alpha)$, where $r:[0, \alpha] \rightarrow \mathbb{R}_+$ is a reputation loss function, and $\theta > 0$ is an exogenous scaling factor.

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We assume that $r(\bar{\alpha})=0$ and by extension $r(\alpha)=0$ for $\alpha > \bar{\alpha}$. We also assume that the reputation loss function is decreasing in α ($r'<0$) for $\alpha < \bar{\alpha}$. The idea is that everybody eventually learns α and it is reasonable to assume that no leader loses his reputation for endorsing a project for which contributions are revealed to be individually rational ex-post (projects with $\alpha \geq \bar{\alpha}$). It is also reasonable to assume that a leader who endorses higher quality projects is less likely to lose his reputation ($r'<0$ for $\alpha < \bar{\alpha}$).

At the end of the first stage followers observe the leader's endorsement but are unable to observe his contribution. This is a key assumption which prevents full revelation of the information.

In the second stage of the game, having observed the leader's endorsement decision, followers update their beliefs about α and simultaneously decide how much to contribute to the public project. A follower's strategy is $X_f: \{0,1\} \rightarrow \mathbb{R}_+$.

An equilibrium of the leader-follower game is a strategy profile (X_0^*, X_f^*, D^*) such that X_0^* maximizes the leader's payoff for any $\alpha \in [0, \bar{\alpha}]$; X_f^* maximizes follower f's expected payoff given the leader's endorsement strategy; and $D^*(\alpha) = 1$ if the leader's gain from endorsement is larger than his cost for any $\alpha \in [0, \bar{\alpha}]$ given the follower's strategies, and $D^*(\alpha) = 0$ otherwise.¹

It is clear that each follower has a unique optimal strategy in equilibrium which depends only on his expectation about α given the leader's endorsement strategy. This means that all followers choose the same equilibrium strategy. Thus, to analyze the equilibria we only focus on a representative follower.

We can distinguish two types of equilibria. One is a trivial equilibrium in which the leader endorses the project with zero probability, and followers never contribute (we will argue- in footnote 14- that this equilibrium is not reasonable). The alternative is an endorsement equilibrium in which the leader endorses the project with positive probability. We show that any

1- The equilibrium concept employed here is a Bayesian Nash Equilibrium in the sense of Crawford and Sobel (1982).

endorsement equilibrium must take a particular form which is stated by Lemma 1. All proofs are in the appendix.

Lemma 1: In any endorsement equilibrium the leader endorses the project ($D^*(\alpha) = 1$) with positive (but less than 1) probability, and follower f makes a positive contribution if and only if the leader endorses the project (that is, $X_f^*(1) > 0$ and $X_f^*(0) = 0$).

We use threshold strategies to characterize the endorsement equilibrium: if the leader adopts the threshold strategy t then he endorses the project if and only if $\alpha > t$. In that case, follower f's optimal strategy in equilibrium depends on the leader's equilibrium threshold t^* , and we write $X_f^*(\cdot; t^*)$ to reflect this dependence. In any endorsement equilibrium with threshold t^* the leader earns $\alpha m X_f^*(1; t^*) - m\theta r(\alpha)$ if she endorses the project and 0 if she does not. Therefore, he adopts a threshold strategy t^*

$$\text{such that: } t^* = \frac{\theta r(t^*)}{X_f^*(1; t^*)} \quad (\text{equation 1}).$$

Recall that follower f's unique equilibrium strategy depends only on his expectation about α given the leader's endorsement strategy. Thus, $X_f^*(1; t^*)$ depends only on $E(\alpha | \alpha > t^*)$. Clearly, $X_f^*(1; t^*) > 0$ iff $E(\alpha | \alpha > t^*) > \alpha = U'(w)$.¹ Define $t_0 \hat{l}(0, \bar{\alpha})$ such that $E(\alpha | \alpha \geq t_0) = \bar{\alpha}$.² Then $X_f^*(1; t^*) > 0$ iff $t^* > t_0$. The following theorem characterizes the endorsement equilibrium.

Theorem 2: There exists a unique endorsement equilibrium. In that equilibrium the leader chooses a threshold strategy $t^* \hat{l}(t_0, \bar{\alpha})$ and every follower makes a positive contribution if the leader endorses the project and makes no contribution otherwise.

Remark 1: Clearly, t_0 is the minimum endorsement threshold that leaves follower f willing to contribute. According to Theorem 2, the leader's

1- In this case, the first order condition implies that $E(\alpha | \alpha > t^*) = U'(w - X_f^*(1; t^*))$.

2- Because $E(\alpha | \alpha > 0) < \bar{\alpha}$, $E(\alpha | \alpha \geq \bar{\alpha}) > \bar{\alpha}$, and since $\partial E(\alpha | \alpha > t^*) / \partial t^* > 0$, such t_0 does exist and is unique.

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endorsement threshold t^* is always higher than t_0 . Because, as t^* approaches t_0 , $X_f^*(1; t^*)$ approaches to 0 and this gradually reduces the leader's endorsement gains and finally eliminates his incentive to endorse a project when $\alpha = t_0$ (and clearly when $\alpha < t_0$). Since $t^* > t_0$, follower f is always willing to make a positive contribution whenever she observes the leader's endorsement. Thus, we claim that the leader is always credible. By our definition a leader is credible if after observing his endorsement the dominant strategy of the followers is to participate in the project. Intuitively, the leader is credible if the discrepancy between his incentives and the incentives of the individual follower is not too large (a condition which is satisfied in our model).

Remark 2: Because $t^* < \bar{\alpha}$, Theorem 2 implies that the unique endorsement equilibrium of the leader-follower game produces positive contributions for strictly more values of α than does the complete information equilibrium. The reason is that leader's exclusive access to α allows him to persuade his followers to participate for values of α at which they would be unwilling to participate if they were fully informed (i.e, for $\alpha < \bar{\alpha}$).¹

Theorem 2 claims only that the leader-follower game produces positive contributions more often than does the complete information game². It does not, however, make any claim about followers' amount of contribution.

The endorsement equilibrium of the leader-follower game yields positive contributions when $t^* < \underline{\alpha} < \bar{\alpha}$, while the complete information equilibrium does not. When $t^* < \underline{\alpha} < \bar{\alpha}$, however, the ex-post comparison becomes more complicated because contributions are positive in both

1- The following argument suggests that the trivial equilibrium is not reasonable and therefore the endorsement equilibrium is the likely outcome of the game. If follower f observes an endorsement decision, then a reasonable argument for him is that the leader must have endorsed a project with $\alpha > t^* > t_0$. Given this it is optimal for follower f to make a positive contribution.

2- The leader-follower setting is in theory unable to eliminate the free-riding problem under symmetric information; this happens because when information is symmetric, followers can effectively free-ride on the leader (See Varian 1992).

scenarios but they are of different magnitudes. The optimal contribution under complete information depends on the true value of α but the followers' optimal contribution in the leader-follower scenario depends on $E(\alpha \hat{E} \alpha > t^*)$. Therefore, when $t^* < \alpha < \alpha$, the endorsement equilibrium of the leader-follower game does not improve cooperation ex-post unless $E(\alpha \hat{E} \alpha > t^*) > \alpha$.¹ Theorem 3 provides an ex-ante comparison.

Theorem 3: For θ sufficiently large, in an endorsement equilibrium,

$$E_{\alpha} [X_f^*] > E_{\alpha} [X(\alpha)].$$

According to Theorem 3, if the cost of a bad endorsement is large enough (if θ is sufficiently large), then expected contributions will be higher ex-ante in the leader-follower setting than under complete information.

Theorem 3 simply shows that, if preventing full revelation of information does not improve the followers' cooperation ex-post in some states, it improves them ex-ante (on average) if the leader's cost of a bad endorsement is sufficiently large.

The following provides a numerical example which satisfies all the structural assumptions of the basic model. Suppose that the utility function of each player has the following form: $\alpha \sum x_i + \ln(1.25 - x_i)$, where α is uniformly distributed on the interval $[0,1]$. Under the complete information model, $X(\alpha) = 0$ for $\alpha \leq 0.8$ and $X(\alpha) = 1.25 - 1/\alpha$ for $\alpha > 0.8$.² A simple expected value calculation shows that the expected (ex-ante) contribution under complete information is $E_{\alpha} [X(\alpha)] = 0.0269$. Now consider the leader-follower model. According to Theorem 2 and footnote 12, $X_f^* = 0$ for $\alpha \leq t^*$ and $X_f^* = 1.25 - 1/E(\alpha \hat{E} \alpha > t^*)$ for $\alpha > t^*$, where $t^* \in (0.6, 0.8)$.³ Theorem 3 simply states that if t^* is not too small (if the leader's cost of a bad endorsement is sufficiently large), the leader-follower model improves followers' cooperation ex-ante (on average). To confirm this suppose that $t^* = 0.8$; then a simple conditional expected value calculation shows that E_{α}

1- The leaders' binary decision has one weakness: sometimes the leader is unable to credibly transmit his useful information to his followers. Theorem 3 shows, however, that in spite of this shortcoming the leader's binary signal can still on average (ex-ante) improve cooperation.

2- Recall that $0.8 = U'(w) = 1/w = 1/1.25$.

3- Recall that $0.6 = t_0$.

$[X_f^*] = 0.02778 > E[X(\alpha)] = 0.0269$ which means that the claim made by Theorem 3 holds. Now reduce t^* to $0.76 \in (0.6, 0.8)$. Then $E[X_f^*] = 0.02727 > E[X(\alpha)] = 0.0269$ which still supports the validity of Theorem 3.¹

4- Conclusion

This article develops a group project which exhibits the standard free-riding problem. It argues that concentrating information in the hands of a leader and preventing full revelation can solve the free-riding problem which exists under full revelation or complete information. To do so, we analyze two different scenarios: A complete information scenario in which all players are informed about the project quality and simultaneously decide how much to contribute, and an incomplete information setting in which only one player (a leader) is informed about the project quality and is unable to partially reveal his information to his ignorant followers.

We show that by preventing full revelation of project quality, followers are induced to cooperate more than they would were the leader's signal to reveal project quality fully.² We show that if preventing full revelation of information does not improve the followers' cooperation ex post in some states, it improves them ex ante (on average) if the leader's cost of a bad endorsement is sufficiently large.

This paper provides a motive for concentrating information only in the hands of a single leader instead of simply making information available to everyone. Ignorance thus can improve cooperation³.

1- This numerical example has been repeatedly replicated by the author. More details are available upon request.

2- At least in some states.

3- This model is appropriate for single-shot and not repeated collective actions (such as task forces which are temporary units, or ad hoc committees established to work on a single-shot collective activity). One interesting extension of this paper is to address the effectiveness of our leadership theory in a repeated collective action game.

Appendix

Lemma (A): Let g and f be continuous functions of x and suppose $f(x) < g(x)$ for all $x \geq x_0$. Then there exists $x < x_0$ such that $f(x) < g(x)$.

Proof: Suppose by way of contradiction that no such x exists; that is $f(x) \geq g(x)$ for all $x < x_0$. Let $\{x_n\}$ be a sequence that converges to x_0 from the left. Then continuity of g and f implies that $f(x_n) - g(x_n)$ converges to $f(x_0) - g(x_0)$ from the left. Then, $f(x_n) - g(x_n) < f(x_0) - g(x_0) < 0$; a contradiction.

Proof of Lemma 1:

Since each follower has a unique optimal strategy and all followers choose the same strategy in equilibrium, we study only one representative follower. Let's simplify follower f 's strategies by dividing them into the four following categories.¹

PP category: follower f always chooses $X_f^* > 0$.

NN category: follower f always chooses $X_f^* = 0$.

M category: follower f chooses $X_f^* > 0$ if the leader endorses the project and $X_f^* = 0$ otherwise.

R category: follower f chooses $X_f^* = 0$ if the leader endorses the project and X_f^* otherwise.

The proof has four steps.

(i) *Follower f does not choose PP.*

Because by assumption $E(\alpha) < U'(\omega)$. This implies that PP is strictly dominated by NN.

(ii) *The probability that the leader endorses the project is neither zero nor one.*

Suppose that the leader endorses the project with probability one. Then M is equivalent to PP and is strictly dominated because of (i), implying that follower f chooses R or NN. This implies that follower f chooses $X_f^* = 0$. Given this the leader should not endorse the project all the time because his payoff from endorsement is negative whenever $\alpha < \omega$ (Because the leader gains nothing from his endorsement given $X_f^* = 0$ but pays a positive endorsement cost whenever $\alpha < \omega$): a contradiction.

1- This of course does not mean that follower f has only four strategies.

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Suppose that the leader endorses the project with probability zero. Then R is equivalent to PP and is strictly dominated, implying that follower f chooses M or NN. This implies that follower f chooses $X_f^*=0$. This is the trivial equilibrium.

(iii) Follower f does not choose NN.

Suppose that follower f chooses the NN type strategy. If the leader endorses with probability zero, then this is the trivial equilibrium. If the leader endorses with positive probability, then this can only be for a subset of α such that $\alpha \geq \bar{\alpha}$, because for $\alpha < \bar{\alpha}$ positive endorsement cost implies that endorsement is not optimal. In this case, choosing NN is not optimal for follower f because his expected gain from contribution is positive if α is expected to be larger than $\bar{\alpha}$.

(iv) Follower f does not choose R.

Suppose that follower f chooses the R type strategy. Then the leader should never endorse the project (contradicts ii).

Proof of Theorem 2:

Recall from page 10 that in any endorsement equilibrium the leader adopts a threshold strategy t^* such that $t^* m X_f^*(1; t^*) - m\theta r(t^*) = 0$. Also recall from Lemma 1 that in any endorsement equilibrium follower f chooses the M type strategy meaning that she chooses $X_f^* > 0$ if the leader endorses the project and $X_f^* = 0$ otherwise. Recall $t_0 < \bar{\alpha}$. To prove this theorem, it is sufficient to show that:

(i) There exists a unique $t^* \hat{l}(t_0, \bar{\alpha})$ such that $h(t^*) = t^* m X_f^*(1; t^*) - m\theta r(t^*) = 0$. This simply proves the uniqueness of the equilibrium as claimed by the theorem.

(ii) If followers choose M, then the unique optimal endorsement strategy for the leader is the threshold strategy t^* . This simply means that the leader's optimal strategy is to endorse projects that have a marginal return greater than t^* knowing that the optimal strategy of the followers is to mimic him.

(iii) If the leader chooses the threshold strategy t^* , then M is optimal for follower f. This simply means that followers' optimal strategy is to follow the leader given that they know that the leaders' optimal strategy is to endorse projects with a marginal return higher than t^* .

Parts (ii) and (iii) simply follow the usual way of proving that a strategy profile constitutes a Nash Equilibrium (In this case a Bayesian Nash Equilibrium).

Proof of (i):

Fact (1): $h(t_0) < 0$ because $X_f^*(1; t_0) = 0$ and $r(\alpha) > 0$ for $\alpha < \bar{\alpha}$.

Fact (2): $h(\alpha) > 0$ because $X_f^*(1; \alpha) > 0$ and $r(\alpha) = 0$ for $\alpha \geq \bar{\alpha}$.

Fact (3): $h(\cdot)$ is continuous and increasing in t^* (because $r' < 0$, and clearly $X_f^*(1; t^*)$ is increasing in t^*).¹

Facts 1, 2 and 3 imply (i).

Proof of (ii):

The leader's endorsement decision yields α in $X_f^*(1; t^*) - m\theta r(\alpha)$ if he endorses the project and 0 otherwise. Therefore, using (i) Leader's equilibrium condition reduces to: endorse the project if $\alpha > t^*$ and not to endorse the project otherwise. Hence the unique optimal endorsement strategy for the leader is the threshold strategy t^* .

Proof of (iii):

Recall that, M is optimal for follower f iff $E(\alpha \hat{=} \alpha > t^*) > \bar{\alpha}$ and $E(\alpha \hat{=} \alpha \leq t^*) < \bar{\alpha}$. Since $\alpha > t^* > t_0$, $E(\alpha \hat{=} \alpha \leq t^*) < \bar{\alpha}$, and $E(\alpha \hat{=} \alpha > t^*) > \bar{\alpha}$. Therefore, M is optimal for follower f if the leader chooses the threshold strategy t^* .

Proof of Theorem 3:

Definition of $X(\alpha)$ implies that under complete information: $\alpha = U'(w - X(\alpha))$ for all $\alpha \geq \bar{\alpha}$ (Statement i). We also know that under incomplete information (See footnote 12): $E(\alpha \hat{=} \alpha > t^*) = U'(w - X_f^*(1, t^*))$ (Statement ii).

Since $U''' > 0$, Jensen's inequality implies that: $U'[E(Z)] < E[U'(Z)]$ for any random variable Z . Therefore, $U'[\underset{\alpha}{E}(w - X(\alpha) \hat{=} \alpha \geq \bar{\alpha})] < \underset{\alpha}{E}[U'(w - X(\alpha) \hat{=} \alpha \geq \bar{\alpha})]$ (Statement iii).

1- Because, $X_f^*(1; t^*)$ increases when $E(\alpha \hat{=} \alpha > t^*)$ goes up (see footnote 12), and $E(\alpha \hat{=} \alpha > t^*)$ increases when t^* goes up.

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Statements (i) and (iii) imply that: $U'[\underset{\alpha}{E} (w-X(\alpha) \hat{=} \alpha \geq z)] < \underset{\alpha}{E}$ ($\alpha \hat{=} \alpha \geq z$) at $z = \bar{\alpha}$. Since α is distributed continuously, both sides of the above inequality are continuous in z . Therefore, Lemma A implies that there exists $T\hat{l}(t_0, \bar{\alpha})$ such that $U'[\underset{\alpha}{E} (w-X(\alpha) \hat{=} \alpha > T)] < \underset{\alpha}{E} (\alpha \hat{=} \alpha > T)$. Recall that $t^* = \theta r(t^*)/X_f^*(1, t^*)$, and from Theorem 2 that $t^*\hat{l}(t_0, \bar{\alpha})$. Choose θ^* such that $t^* = T\hat{l}(t_0, \bar{\alpha})$. Because $\partial E(\alpha \hat{=} \alpha > t^*)/\partial t^* > 0$, and since $\partial t^*/\partial \theta > 0$ we have: $U'[\underset{\alpha}{E} (w-X(\alpha) \hat{=} \alpha > t^*)] < \underset{\alpha}{E} (\alpha \hat{=} \alpha > t^*)$ for all $\theta \geq \theta^*$.¹

Using (Statement ii) we have: $U'[\underset{\alpha}{E} (w-X(\alpha) \hat{=} \alpha > t^*)] < U'(w - X_f^*(1, t^*))$. From $U'' < 0$ it follows that: $w - \underset{\alpha}{E} [X(\alpha) \hat{=} \alpha > t^*] > w - X_f^*(1, t^*)$ or $\underset{\alpha}{E} [X(\alpha) \hat{=} \alpha > t^*] < X_f^*(1, t^*)$. Thus, $\Pr(\alpha > t^*) \cdot \underset{\alpha}{E} [X(\alpha) \hat{=} \alpha > t^*] < \Pr(\alpha > t^*) \cdot X_f^*(1, t^*)$ (Statement iv).

Recall that $t^*\hat{l}(t_0, \bar{\alpha})$. Thus we have: $\underset{\alpha}{E} [X(\alpha) \hat{=} \alpha \leq t^*] = 0$. Thus, $\underset{\alpha}{E} [X(\alpha)] = \Pr(\alpha \leq t^*) \cdot \underset{\alpha}{E} [X(\alpha) \hat{=} \alpha \leq t^*] + \Pr(\alpha > t^*) \cdot \underset{\alpha}{E} [X(\alpha) \hat{=} \alpha > t^*] = \Pr(\alpha > t^*) \cdot \underset{\alpha}{E} [X(\alpha) \hat{=} \alpha > t^*]$ (Statement v).

From (iv) and (v) we have: $\underset{\alpha}{E} [X(\alpha)] < \Pr(\alpha > t^*) \cdot X_f^*(1, t^*)$. Thus, $\underset{\alpha}{E} [X_f^*] > \underset{\alpha}{E} [X(\alpha)]$.

1- When θ increases, t^* increases (because $(\partial t^*/\partial \theta > 0)$). The left hand side of the inequality stays the same because $X(\alpha)$ is zero when α varies over the interval $(t_0, \bar{\alpha})$. The right hand side of the inequality, however, increases because $\partial E(\alpha \hat{=} \alpha > t^*)/\partial t^* > 0$. Therefore, if the inequality holds for θ^* , it should also hold for $\theta > \theta^*$.

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