

Derivation of Shadow Price for CO₂ Gas Emission Using Distance Function Approach

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Abstract

The purpose of this study is to estimate the shadow price of CO₂ gas emission utilizing the output distance function including good products (GDP) and bad products (CO₂). At the first, output oriented technical efficiency under the two assumptions of weak and strong disposability of CO₂ and Environmental Efficiency Index are estimated using the Iran's economic data. Shadow price of CO₂ gas emission is derived employing a Translog distance function.

Results show that the average of technical efficiency under the two assumptions of strong and weak disposability and environmental efficiency are equal to 1.0743, 1.0910 and 0.99267, respectively. Average shadow price of CO₂ emission is 0.10933 million US dollars per thousand tons based on the constant price of the year 2000.

Keywords: Environmental Efficiency, Shadow Price of CO₂, Distance Function, Iran.

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1- Introduction

Environmental problems are threatening world's sustainability. At current rates of emission, the accumulation of greenhouse gases in the upper atmosphere is expected to elevate average global surface temperature by approximately 0.3–2.5°C in the next 50 years and 1.4–5.8° C in the next century (Zelek and Shively, 2003). Economic and ecological impacts of global warming are one of the most important debates.

Global climate problems led the international agreement at the Earth Summit in 1992 to reduce greenhouse gas emissions by 2000 to the level of 1990. Afterward, Kyoto Protocol in 1997 under which industrialized countries agreed to legally binding emissions reduction averaging about 5 percent of 1990 levels by period 2008-2012 (Kumar and Khanna, 2002). In Kyoto Protocol, no numerical targets for emissions of developing countries, like Iran, were set. But on one hand, the cost of air pollution is 1.6 percent of GDP and on the other hand, the cost of environmental degradation is about 10000 million US\$ annually and close to 8.8 percent of Iran's GDP (World Bank, 2005). Also environmental problems are not only matters of regional significance, but have taken on global significance. Therefore, studying the negative impacts of pollution emission on economic growth make sense for Iran.

To reduce these negative impacts, it is necessary to regulate pollution emission. Estimating marginal cost of abatement will help the country analyze the cost effectiveness of alternatives ways to reduce emission, predict environmental effects of economic growth, and make decision about trading international tradable pollution permit.

The most important greenhouse gas, and the one has received the most attention is Carbon Dioxide (Zelek and Shively, 2003; Zaim and Taskin, 2000; Tahara et al, 2005).

This paper uses the nonparametric DEA (Data Envelopment Analysis) approach (Fare and Grosskopf, 2000) to estimate technical and environmental efficiencies of Iran economy; then employs a distance function to generate shadow price for CO₂ defined as the marginal cost of abatement in terms of desirable output (GDP).

2- Methodology

2.1. The Technology Set

Suppose a country is a decision making unit (DMU). This DMU employs a vector of inputs $x (=x_1, x_2, \dots, x_K)$ to produce a vector of desirable output $y (=y_1, y_2, \dots, y_M)$ and a vector of undesirable outputs $b (=b_1, b_2, \dots, b_N)$, where K, M and N are non-negative. Let $P(x)$ be the production possibility set for a given input vector x and $L(y, b)$ is the input requirement set for a given level of output (y, b) . The technology set is defined as (Fare and Grosskopf, 2000):

$$T = \left\{ (y, b, x) \in R^{M+N+K}, (y, b) \in P(x), x \in L(y, b) \right\} \quad (1)$$

Now, suppose the desirable (good) and undesirable (bad) outputs are null-joint; that is, the production of good outputs are accompanying with bad outputs production, thus if $(y, b) \in P(x)$ and $y = 0$ then $b=0$.

There are two assumptions about the disposability characteristic of bad outputs that if they are strongly (freely) to dispose or weakly dispose. Strong disposability means that the disposition of bad outputs is not costly (Fare et al, 1993).

If

$$(y, b) \in P(x) \text{ and } (y', b') \leq (y, b) \Rightarrow (y', b') \in P(x) \quad (2)$$

Equation (2) implies that if an output set is feasible with a given level of inputs then every other output set smaller than it is feasible with the same level of inputs. But weak disposability of bad outputs says that the DMU have to divert the resources from good production to dispose of bad outputs, so the disposition of bad outputs is costly (Fare et al, 1993).

If

$$(y, b) \in P(x) \text{ and } 0 \leq \theta \leq 1 \Rightarrow (\theta y, \theta b) \in P(x) \quad (3)$$

Using the above notations and assuming that the production process satisfies strong disposability of both outputs (good and bad outputs) and

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inputs, the constant returns to scale (CRS) output set $P(x|C,S)$ can be constructed by means of

$$P(x|C,S) = \left\{ (y,b) : \sum_{t=1}^T z_t y_{tM} \geq y_t, \sum_{t=1}^T z_t b_{tM} \geq b_t, \sum_{t=1}^T z_t x_{tN} \leq x_t, z \in R_+^T \right\} \quad (4)$$

Where C and S referee to constant returns to scale and strong disposability of bad output, respectively and $z (=z_1, z_2 \dots z_T)$ is the vector of intensity variable (Zaim and Taskin, 2000). Similarly, a CRS technology satisfying the weak disposability of undesirable outputs and strong disposability of desirable outputs and inputs can be represented as an output set as shown below (Zaim and Taskin, 2000):

$$P(x|C,W) = \left\{ (y,b) : \sum_{t=1}^T z_t y_{tM} \geq y_t, \sum_{t=1}^T z_t b_{tM} = b_t, \sum_{t=1}^T z_t x_{tN} \leq x_t, z \in R_+^T \right\} \quad (5)$$

where W referees to weak disposability of bad output.

Equations (4) and (5) construct a best practice technology from the observed data on inputs and outputs relative to which technical efficiency of the production of each year can be calculated.

There are two alternatives of technical efficiency: input oriented and output oriented measures of technical efficiencies and there are two approaches to measure it: parametric and nonparametric. Stochastic Frontier Analysis (SFA) is used to estimate technical efficiency in parametric approach and Data Envelopment Analysis (DEA) in nonparametric. In this paper, we want to calculate the output oriented measure of technical efficiency using DEA that can be shown as below (Fare and Grosskopf, 2000):

$$F(y,b,x) = \max\{\theta : \theta(y,b) \in P(x)\} \quad (6)$$

Where θ is efficiency index. The reciprocal of this index shows the output distance from production frontier, and defined as (Kumar and Khanna, 2002):

$$D(y, b, x) = \min \left\{ \theta : \left(\frac{(y, b)}{\theta} \right) \in P(x) \right\} \quad (7)$$

The distance function, defined by Shepherd (1970), equal to one shows that the DMU operates at the production frontier and when the DMU operates below the frontier, distance function is smaller than one but greater equal than zero. The output distance function has three advantages (Fare et al, 1993): first, it allows to model bad outputs; second, it can describe completely the technology and models joint production of multiple outputs; third, the duality between distance and revenue function allows generating shadow prices of outputs. The output distance function measures the maximum proportion by which all outputs increased proportionally when the inputs are held constant but reduction in bad outputs will divert inputs to good outputs. Therefore, the output distance function should be non-decreasing in good outputs and non-increasing in bad outputs and inputs.

2-2- Environmental Efficiency

The output distance function can be used to estimate the environmental efficiency (EE) with respect to bad output. From Zaim and Taskin (2000) EE is the ratio of the square root of the output distance function under strong disposability of bad output to the square root of the output distance function under weak disposability of bad output (Kumar and Khanna, 2002).

$$EE = \frac{\sqrt{D(y^t, b^t, x^t | C, S)}}{\sqrt{D(y^t, b^t, x^t | C, W)}} \quad (8)$$

Environmental efficiency takes the values in the range of 0-1. EE equal to one shows that $D(y^t, b^t, x^t | C, S) = D(y^t, b^t, x^t | C, W)$. That is the disposition of bad output is free and potential loss of good output under weak disposability of bad output is zero. Coelli et al (2006) and Murty et al (2005) used of equation (8) to calculate environmental efficiency.

2-3- Shadow Price of Bad Output

Pittman (1983) employed a profit function to estimate the shadow price of bad outputs. But Fare et al (1993) introduced an alternative way. They showed how the output distance function can be used to generate shadow prices of bad outputs. They estimated an output distance function and employed a dual Shepherd's lemma to calculate shadow price of outputs both for marketable and no marketable outputs. As noted by Fare et al (1993) "the ratio of any two of these output shadow prices reflects the relative opportunity cost of those outputs, i.e., they are equivalent to the marginal rate of transformation". The shadow price of bad output calculated by means of this method reflects the marginal cost of abatement and the trade-off between bad and good outputs. The shadow prices of bad (r_b) outputs can be calculated as follows (Fare et al, 1993):

$$r_b = r_y \times \frac{\frac{\partial D(y^t, b^t, x^t | C, W)}{\partial b}}{\frac{\partial D(y^t, b^t, x^t | C, W)}{\partial y}} \quad (9)$$

where r_y is the price of good output.

2-4- Estimation Method and Data

In order to generate the shadow price of bad output, we need to parameterize and calculate the parameters of the output distance function. In this paper, we employed a translog form for output distance function to model the technology. This functional form has the advantage of flexibility that is more important for our purpose. Fare et al (1993), Coggins and Swinton (1996) and Fuentes et al (2001) used this functional form to estimate the output distance function. This functional form is as below:

$$\begin{aligned}
 \ln D^t(y^t, b^t, x^t | C, W) = & \alpha_0 + \sum_{k=1}^K \alpha_k \ln x_k^t + \frac{1}{2} \sum_{k=1}^K \sum_{k'=1}^K \alpha_{kk'} \ln x_k^t \ln x_{k'}^t \\
 & + \sum_{m=1}^M \beta_m \ln y_m^t + \frac{1}{2} \sum_{m=1}^M \sum_{m'=1}^M \beta_{mm'} \ln y_m^t \ln y_{m'}^t + \sum_{n=1}^N \gamma_n \ln b_n^t \\
 & + \frac{1}{2} \sum_{n=1}^N \sum_{n'=1}^N \lambda_{nn'} \ln b_n^t \ln b_{n'}^t + \sum_{k=1}^K \sum_{m=1}^M \delta_{km} \ln y_m^t \ln x_k^t + \sum_{k=1}^K \sum_{n=1}^N \varphi_{kn} \ln b_n^t \ln x_k^t \\
 & + \theta_t \times T + \frac{1}{2} \theta_{tt} \times T^2 + \sum_{m=1}^M \theta_{mt} \ln y_m^t \times T + \sum_{n=1}^N \theta_{nt} \ln b_n^t \times T + \sum_{k=1}^K \theta_{kt} \ln x_k^t \times T
 \end{aligned} \tag{10}$$

To estimate the functional form (10), it must be imposed the restrictions on symmetry and homogeneity. Equation (11) and (12) below show the homogeneity degree 1 of output distance function in outputs and the symmetry condition, respectively.

$$\sum_{m=1}^M \beta_m + \sum_{n=1}^N \gamma_n = 1 \tag{11}$$

$$\begin{aligned}
 \sum \alpha_{kk'} &= \sum \beta_{mm'} = \sum \gamma_{nn'} = \sum \delta_{km} = \sum \varphi_{kn} = 0 \\
 \alpha_{kk'} &= \alpha_{k'k} \\
 \beta_{mm'} &= \beta_{m'm} \\
 \gamma_{nn'} &= \gamma_{n'n}
 \end{aligned} \tag{12}$$

The parameters of functional form (10) estimated using the parametric approach.

Data on gross domestic product (GDP), as a good output, CO2, as a bad output and total labor force (L), capital stock (K), and commercial energy consumption (E) as inputs used in this paper are collected from the World Development Indicators(World Bank 2005). GDP and capital stock are measured in 1995 US dollars, whereas CO2 emission and energy consumption are measured in thousand metric tones and total labor force is measured in millions of employees.

3- Results

3-1- The output distance function

In table 1, estimated parameters for the output distance function under weak disposability of CO2 are presented. The first derivative of the model with respect to all input variables (except for capital), time variable and undesirable output (CO2 emission) have negative signs but have positive sign with respect to GDP. The negative signs on inputs indicate that output distance function value is non-increasing with respect to inputs and economy becomes closer to the frontier.

Coefficients of the time variables indicate the presence of technological progress in the regression model. A negative sign for all of time variables shows technological progress but positive sign for that shows technological deterioration. In this study, except for time variable and interaction term between capital and time, all time variables have negative signs. On the other hand, negative sign of first order derivative of distance function with respect to time shows technological progress.

Table 1: Parameter Estimates of Output Distance Function

Variable	Value	T-Ratio	Variable	Value	T-Ratio
INTERCEPT	-0.64	-0.58	ENERGY×CAPITAL	-0.32	-0.91
GDP	6.67	2.28	ENERGY×LABOR	-4.22	-4.83
GDP×GDP	5.70	4.02	CAPITAL	13.13	1.91
GDP×CO2	-6.55	-11.16	CAPITAL×CAPITAL	0.19	1.07
GDP×ENERGY	4.86	4.02	CAPITAL×LABOR	-1.07	-2.85
GDP×CAPITAL	-0.31	-0.72	LABOR	-13.43	-2.93
GDP×LABOR	-2.43	-3.68	LABOR×LABOR	4.43	4.27
CO2	-5.67	-1.94	TIME	1.84	2.67
CO2×CO2	4.58	8.50	TIME×TIME	0.01	2.13
CO2×ENERGY	-1.71	-2.59	TIME×CAPITAL	0.03	1.04
CO2×CAPITAL	0.76	4.67	TIME×LABOR	-0.06	-1.14
CO2×LABOR	2.08	3.97	TIME×ENERGY	-0.11	-2.91
ENERGY	-0.94	-0.91	TIME×CO2	0.01	0.82
ENERGY×ENERGY	3.76	3.73	TIME×GDP	-0.04	0.74
R-Square=0.98					

3-2-Environmental Efficiency

An increase in GDP along with CO₂ reduction means efficiency gain in environmental terms. Recall that the output distance function can be used to calculate environmental efficiency. If the values of distance function under two assumption of weak and strong disposability of CO₂ are the same then EE equal to one and CO₂ emission can be abated without reducing GDP; but if they are different, country has an environmentally binding production technology. Figure 1 shows trends for the value of technical efficiency under two assumptions of weak and strong disposability of CO₂. The figure also shows environmental efficiency of Iran economy for period 1974-2004. Table 2 presents mean values for output distance function under weak and strong disposability and environmental efficiency, too. Except for most years of the study, environmental efficiency is less than one (figure 1) and also average environmental efficiency is less than one (table 2) that reflects Iran has an environmentally binding production technology. The absolute GDP loss due to weak disposability of CO₂ can be calculated as $GDP \times (1-EE)$. Therefore, on average, GDP loss is equal to 1158 millions of 2000 US dollar to use the best practice technology.

Table 2: Mean Output Distance Function and Environmental Efficiency

	Value of distance function		Environmental efficiency	GDP (Millions of 2000 US dollar)
	Weak disposability	strong disposability		
mean	0.93449	0.92068	0.99268	79394
SD	0.05902	0.06215	0.02109	19107

Technical efficiency is the reciprocal of distance value (Coggins and Swinton, 1996), so on average technical efficiency under weak and strong disposability of CO₂ are 1.0701 and 1.0861, respectively. It means technical efficiency value of 1.0701 under weak disposability of CO₂ indicated that it is technologically feasible for Iran to increase its GDP by 7.01 percent and simultaneously decrease the level of CO₂ emission by 7.01 percent to use the best practice technology.

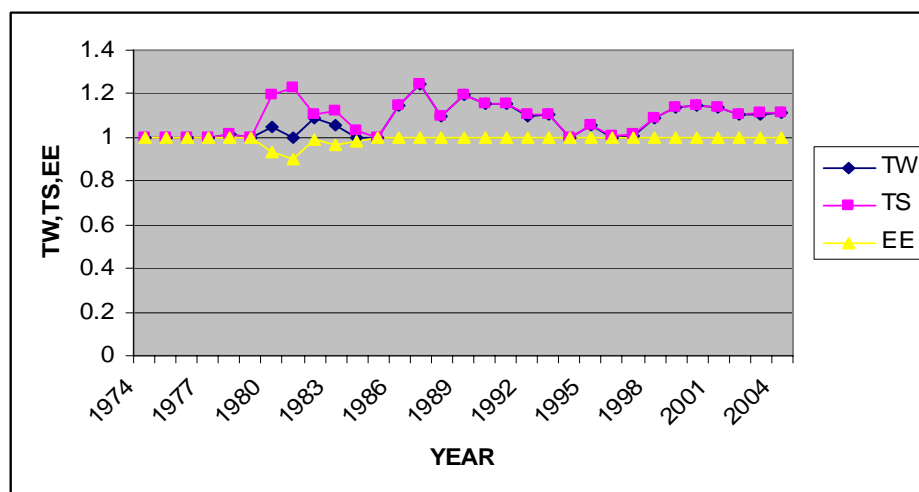


Figure (1): Trends for Technical Efficiency under two assumptions of Weak and Strong Disposability of CO2 and Environmental Efficiency

TW= Technical Efficiency under Weak Disposability of CO2

SD= Technical Efficiency under Strong Disposability

EE= Environmental Efficiency

3-3-Shadow Price of CO2

Our results show that the average shadow price for decreasing of CO2 emission is \$ 0.10933 millions per thousands metric tons (\$109.33 per ton of CO2) at 2000 price.

Recall the estimation of shadow price of CO2 is suitable for decision making about predicting environmental effects of economic growth and trading international tradable pollution permits.

3-3-1- Predicting Environmental Effects of Economic Growth

As noted in section 2.3, shadow price of CO2 can be interpreted as marginal cost of abatement CO2 with respect to GDP and that shows a short term changes in environmental quality if GDP changes while technical efficiency remains constant.

Iranian GDP is US\$127075.15 millions at 2000 prices, and CO2 emission is 393385.39 thousands tons for year 2004. To clarify

environmental effects of economic growth, suppose as an example, Iran experiences GDP growth of %1 or US\$ 1270.7515 millions, while efficiency remains constant. Shadow price, tells us that such increase in GDP results 11623.08 ($=127075.15/0.10933$) thousands tons or 2.95% increase in CO2 emission compared to the original value.

3.3.2. International Tradable Pollution Permit Trade

Since, first, Iran is not one of the members of Kyoto Protocol, and doesn't have to reduce CO2 emission; second, trading international pollution permits is only possible for countries that have agreed to quotas, predominately the OECD countries; third, this study is not cross-country analysis; therefore, trading pollution permits is not applicable to this study. But we present an example to explain how Iran can use shadow price of CO2 for trading pollution permits.

Imagine a simplified version of Kyoto Protocol: two countries, Iran and an industrialized country such as Japan, enter an agreement to hold their total CO2 emission level constant. At this, suppose each country needs to emit more than what they do today. It should negotiate on the price with the partner and purchase a permit to emit additional pollution, while the other country should shrink its pollution by the amount sold.

Assume that technology remains constant, but countries face positive exogenous shock leading to real output growth. Output growth leads to emission increase *ceteris paribus*. Suppose for example, Japans' shadow price for CO2 is \$1519.80 per ton at 2000 price, according to our estimates, Japan agrees to pay at most \$1519.80 for a right to increase its CO2 emission by 1 ton. In turn, Iran agrees to decrease its CO2 emissions by 1 ton if it is paid at least \$109.33. Since it is clearly in the interests of Japan to pay between \$109.33 and \$1519.80 to Iran to buy 1 CO2 emission permit, therefore Iran will shrink its GDP and CO2 emissions, while Japan will expand both GDP and CO2 production; both countries will gain from such transaction *ceteris paribus*. Therefore, the absolute value of shadow price of CO2 increases for Iran and decreases for Japan. If both countries face the perspective of economic growth, the trade of permits will continue until shadow prices across countries equalize.

4- Conclusions

The main purpose of this paper is to estimate shadow price of CO2 emission in Iran. Using time series data on Iran's economy for the period of 1974-2004, technical efficiency under weak and strong disposability of CO2 are estimated by DEA (Data Envelopment Analysis) approach. Then output distance function that incorporates both desirable (GDP) and undesirable (CO2) outputs is estimated. The estimation of distance function is used to generate environmental efficiency index. But distance function estimation under weak disposability of CO2 is only used to derive shadow price of CO2.

Results show that Environmental efficiency (EE) and technical efficiency under the two assumptions are equal to one for most years reflects that Iran's production technology satisfies strong disposability of CO2. It means Iran's economy lies on production frontier; but for years that economy is away from production frontier, country has promising long-term perspective in simultaneous increase of GDP and decrease of CO2, should Iran improves the technology in use.

Our first illustrative example, based on the estimation of the shadow prices, showed how our estimates can be used in forecasting environmental outcomes of economic growth within a short run. Also, estimates of the efficiency allow us to conclude about technological feasibility and scale of potential simultaneous increase of GDP and decrease of environmental pollution.

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