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Optimization of the Competitive Model of the Government and Tax payer under the condition of the possible Existence of Error in Auditing the Tax payers Income tax Report

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Abstract

In the present paper we have analyzed an important problem in the field of game theory, which is faced more frequently in the issues of income tax and has not been paid attention by scientific resources. The problem in question is the affect of taxpayer's unintentional error on optimum auditing strategy. One of the important problems in today's tax organizations is the presence of taxpayers' unintentional error in their future income calculation and anticipation of their own economic activities for reporting on income tax statement. These errors will affect the optimum control and inspection strategy of net overall taxes.

In the presented model in this paper, we have tried to consider the problem in specific condition in which the outcome results are very interesting.

Keyword: Unintentional error; Optimum auditing, auditing strategy

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1- Introduction

In this article we have considered the subject of an error in income tax auditing in the context of game theory. Using the game theory model to analyze income tax auditing, and determining government optimum auditing strategy is one of the scientific resources goals. The most important problem which internal Revenue Service faces is the presence of an error in the tax payer income tax report. This error has a major impact on the optimum control and inspection strategy of the over all net government collected taxes.

The model used in this paper will analyze this effect under specific conditions which result in an unique out come.

2- The Model

In this model the taxpaying group will be analyzed with an income random value *I*, probability function $\rho(I)$, and density function $\mu(I)$ which $I \in [0,\infty)$. We assume that the tax payer evaluates and proclaims his income as I_e value in his self statement tax report, where $I_e = nI$ and $n \in [0,1]$ is a random value with a probability function $m_{(n)}$, and *I* is the real income. Therefore we will have $I_e \in [0,I]$. If n $=1$, then the tax – payer evaluates his income as a real income, and if $n = o$, he has the maximum error value in his evaluation. It is assumed in this paper which the taxpayer always considers the reported income I_d to he greater than the evaluated income I_e. This means $I_d \leq I_e$, or on the other hand the taxpayer considers the reported.[2,3]

We consider the tax that should be paid by the tax payer income $I_d \in [0, I_e]$, to minimizes the real tax valu, with *I* income to be in the form of $\mathbf{T(I) = tI}$ (t is the income tax factor)[1,5,6]. The tax revenue service audits the tax payer's tax report with probability $\mathbf{p(I_d)}$, since it is possible that he does not proclaim his real income accidentally, or intentionally. This audit always determines the real taxpayer's income. A monetary fine is determined for a false claim of $I > I_d$ from the fine function $\mathbf{F(I, I_d) = (t + f)(I - I_d)}$, which is levied for the total paid taxes. In every income tax audition, the government income depends on the income taxes and the monetary fines for the total reported income, although the tax payer corrupts the tax report intentionally, or evaluates and reports unintentionally an incorrect income tax statement report. [8,9]

3- Main Theorems

We are going to start with exploring the main theorems developed for this subject as follows:

Theorem 3.1 *The real income tax of the taxpayer, who has evaluated his income as I_e but his reported income is equal to I_d is calculated from the following mathematical relation*

$$G(I_d, I_e, p(I_d)) = tI_d + p(I_d)[(t+f)\int_0^1 \pi_n(I_e)(\frac{I_e}{n} - I_d)dn] = tI_d + p(I_d)(t+f)(I_{cp}(I_e) - I_d),$$

where $I_{cp}(I_e) = \int_0^1 \pi_n(I_e)\frac{I_e}{n}dn$ *and* $\pi_n(I_e) = \frac{\frac{m(n)}{n}\rho(\frac{I_e}{n})}{\int_0^1 \frac{m(n)}{n}\rho(\frac{I_e}{n})dn}$, $n \in [0,1]$.

Proof: In general the real tax payer's income tax with an audited income of $G(I_d, I_e, p(I_d)) = tI_d + p(I_d)\theta(I_e, I_d)$, reported income of I_e , reported income of I_d , and an auditing report probability function $p(I_d)$ is stated. In which $\theta(I_e, I_d)$ is the taxes for the difference between the audited and reported income and the tax evasion resulted monetary fine.

$$F_X(x) = P(X \le x) = P(nY \le x) = P(Y \le \frac{x}{n}) = F_Y(\frac{x}{n})$$

If we assume $\frac{x}{n}$ = y, we will have;

$$f_X(x) = \frac{dF_X(x)}{dx} = \frac{dF_Y(\frac{x}{n})}{dx} = \frac{1}{n}\frac{d}{dy}F_Y(\frac{x}{n}) = \frac{1}{n}\frac{d}{dy}F_Y(y) = \frac{1}{n}f_Y(y) = \frac{1}{n}f_Y(\frac{x}{n}).$$

We assume $\alpha(I_e)$ is the possible distribution of random variable I_e, then $\alpha(I_e | n)$ is the conditional probability known for value n. In this case:

$$\alpha\,(I_e \mid N=n)=\frac{1}{n}\rho(\frac{I_e}{n}),$$

$$\alpha\,(I_e)=\int_0^1 m_{(n)}\alpha(I_e \mid N=n)dn=\int_0^1 m_{(n)}(\frac{1}{n}\rho(\frac{I_e}{n}))dn=\int_0^1 \frac{m_{(n)}}{n}\rho(\frac{I_e}{n})dn$$

Using Bayes' theorem for known value *Ie*, possible distribution function for real income $I=\frac{I_e}{n}$, then $n\in[\,o\,,\,1]$ can be written as follows :

$$\alpha\,(I \mid I_e)=\frac{m_{(n)}\alpha\,(I_e \mid N=n)}{\alpha\,(I_e)}=\frac{\frac{m_{(n)}}{n}\rho(\frac{I_e}{n})}{\int_0^1 \frac{m_{(n)}}{n}\rho(\frac{I_e}{n})dn}\overset{def}{=}\pi_{n\,(I_e)}\cdot$$

If $I_d \leq I_e$, then

$$G(I_d, I_e, p_{(I_d)})=tI_d+p_{(I_d)}[(t+f)\int_0^1 \pi_{n\,(I_e)}(\frac{I_e}{n}-I_d)dn]=tI_d+p_{(I_d)}$$
$$(t+f)(I_{cp\,(I_e)}-I_d)$$

In which $I_{cp\,(I_e)}=\int_0^1 \pi_{n\,(I_e)}\frac{I_e}{n}dn$ is the tax payer's average income with an observed income of I_e.

For every probability of auditing strategy $\mathbf{p(I_d)}$, the tax payer's optimum strategy is the solution to the following problem.

$$I_{d\,(I_e)} \longrightarrow \min_{0\leq I_d\leq I_e} G(I_d, I_e, p(I_d))$$

If we represent the auditing cost by c, hence the net government income could be expressed as follows:

$$R(p_{(I_d(I_e))})=\int_0^1 m_{(n)}\{\int_0^{\infty}\{tI_{d\,(nI)}+p_{(I_d(nI))}[(t+f)(I-I_{d\,(nI)})-c]\}\rho_{(I)}dI\}dn$$

$$=\int_0^1 m(n)\{\int_0^\infty \{tI_d(I_e)+p(I_d)[(t+f)(\frac{I_e}{n}-I_d(I_e))-c]\}\rho(I_e/n)(dI_e/n)\}\text{dn}$$

$$=\int_0^\infty \{tI_d(I_e)+p(I_d(I_e))[(t+f)(I_{cp}(I_e)-I_d(I_e))-c]\}\alpha(I_e)dI_e$$

$$=\int_0^\infty [G(I_d(I_e),I_e,p(I_d))-p(I_d)c]\alpha(I_e)dI_e \quad ,$$

So that $G(I_d(I_e),I_e,p(I_d))$ is the final tax which will be paid by the tax payer, with an audited income I_e. The government problem is based on probable optimum strategy $p^*(\cdot)$ which maximizes the preceding integration. Next, we assume the function $I_{cp}(I_e)$ increases uniformly. Solution for total income optimization problem: Note that we are seeking the solution for tax payer and the government's optimization problem in the range $I_d \le I_e$.

Theorem 3.2 Assume the function $p(I_d(I_e))$ is the probable auditing function and $I_d(I_e)$ is the reported income for the auditing income Ie, so that are continuous almost everywhere. In order for the function $I_d(I_e)$ to minimize the tax payer's taxes, it is necessary that this function be satisfied by the following conditions:

1) If $I_d(I_e) < I_e$ in the interval $[I_1, I_2]$, then we will have:

a) $p(I_d(I_e))$ is not an increasing function with respect to I_e,

b) $\dfrac{dG}{dI_e} = p(I_d(I_e))(t+f)\dfrac{dI_{cp}}{dI_e}$.

2) If $I_d(I_e) = I_e$ in the interval $[I_1, I_2]$, then we will have:

c) $p'(I_d(I_e)) \le \dfrac{p(I_d)-\hat{p}}{I_{cp}(I_e)-I_e}$.

d) $\frac{dG}{dI_e} \leq p(I_d)(t+f)\frac{dI_{cp(I_e)}}{dI_e}$.

Proof: 1) If we assume that $I_d(I_e) < I_e$ in the interval $[I_1, I_2]$, then:

$$G(I_{d(I_e)}, I_e, p(I_{d(I_e)}))$$
$$= tI_{d(I_e)} + p(I_{d(I_e)})(t+f)(I_{cp(I_e)} - I_{d(I_e)}).$$

Thus, for very $I_d \leq I_e$, will have

$$tI_{d(I_e)} + p(I_{d(I_e)})(t+f)(I_{cp(I_e)} - I_{d(I_e)}) \leq$$
$$tI_d + p(I_d)(t+f)(I_{cp(I_e)} - I_d) \quad (1)$$

We consider the two values of I_e, I_{e_1}, as $I_e < I_{e_1}$, and since $G(I_{d(I_e)}, I_e, p(I_{d(I_e)}))$ is the audited minimum income tax for I_e, hence

$$G(I_{d(I_e)}, I_e, p(I_{d(I_e)})) \leq G(I_{d(I_{e_1})}, I_e, p(I_{d(I_{e1})})).$$

By rewriting inequality (1), first for the values $I_{d(I_e)}$ and $I_d = I_{d(I_{e_1})}$, and then for the values $I_{d(I_{e_1})}$ and $I_d = I_{d(I_e)}$, we will conclude:

$$G(I_{d(I_e)}, I_e, p(I_{d(I_e)})) = tI_{d(I_e)} + p(I_{d(I_e)})(t+f)(I_{cp(I_e)} - I_{d(I_e)}) \leq$$
$$tI_{d(I_{e_1})} + p(I_{d(I_{e_1})})(t+f)(I_{cp(I_e)} - I_{d(I_{e1})}) \quad (2)$$
$$G(I_{d(I_{e_1})}, I_{e_1}, p(I_{d(I_{e1})})) = tI_{d(I_{e_1})} + p(I_{d(I_{e_1})})(t+f)(I_{cp(I_{e1})} - I_{d(I_{e1})}) \leq$$
$$tI_{d(I_e)} + p(I_{d(I_e)})(t+f)(I_{cp(I_{e1})} - I_{d(I_e)}). \quad (3)$$

It is concluded from (2) and (3) that:

$$p(I_{d(I_e)})(t+f)(I_{cp(I_e)} - I_{cp(I_{e1})}) \leq G(I_{d(I_e)}, I_e, p(I_{d(I_e)})) -$$
$$G(I_{d(I_{e_1})}, I_{e_1}, p(I_{d(I_{e_1})})) \leq p(I_{d(I_{e_1})})(t+f)(I_{cp(I_e)} - I_{cp(I_{e1})}).$$

Because $I_{cp(I_e)}$ is uniformly increasing, the latter inequality can be rewritten as follows:

$$-p(I_d(I_e))(t+f)\Delta I_{cp} \le \Delta G \le -p(I_d(I_{e_1}))(t+f)\Delta I_{cp} \quad . \tag{4}$$

From (4) we will conclude that $p(I_d(I_e)) \ge p(I_d(I_{e_1}))$. This means that the auditing probability function $p(I_d(I_e))$ is non – increasing with respect to I_e.

b) Assume that in (4), the variable I_{e_1}, is approaching to I_e, which means

$$\lim_{I_{e_1}\to I_e} p(I_d(I_{e_1}))(t+f)\frac{\Delta I_{cp}}{\Delta I_e} \le \lim_{I_{e_1}\to I_e} \frac{\Delta G}{\Delta I_e} \le \lim_{I_{e_1}\to I_e} p(I_d(I_e))(t+f)\frac{\Delta I_{cp}}{\Delta I_e} .$$

Because functions $p(I_d(I_e))$ and $I_d(I_e)$ are continuous almost everywhere, therefore

$$\frac{dG}{dI_e} = p(I_d)(t+f)\frac{dI_{cp}}{dI_e} .$$

2) Assume $I_d(I_e) = I_e$ in interval $[I_1, I_2]$.

c) Consequently, $\min\limits_{I_d \le I_e} G(I_d, I_e) = G(I_e, I_e)$ and it is necessary that $\frac{\partial G}{\partial I_d}(I_e, I_e) \le 0$. It is concluded that $p'(I_d(I_e)) \le \dfrac{p(I_d) - \hat{p}}{I_{cp}(I_e) - I_e}$.

d) With calculation differential function $G(I_d, I_e)$ it is concluded that:

$$\frac{dG}{dI_e} = \frac{d}{dI_e}[(\hat{p} - p(I_e))I_e + p(I_e)I_{cp}(I_e)](t+f) = [\hat{p} + (p(I_e) - \hat{p}) + p(I_e)(I'_{cp}(I_e) - 1)](t+f)$$
$$\le (t+f)p(I_e)I'_{cp}(I_e). \qquad \square$$

To express the next theorem, we need to define $\Phi(I_e) = (t+f)(I_{cp}(I_e) - I_e) - c$.

Theorem 3.3 *If* $\Phi(I_e) \le 0 \;\; \forall I_e$, *then in this case, the government optimum strategy is in the form of step function* $p^*(I_d(I_e)) = \begin{cases} \hat{p}, & I_e \le \hat{I} \\ 0, & I_e > \hat{I} \end{cases}$.

Proof: We consider the set $A = \{ I \mid p(I_d) \ge \hat{p}; \quad \forall I_d \le I \}$, assume $\bar{I} = Sup\, A$. If $A = \varnothing$, then we set $\bar{I} = 0$. If $I_e < \bar{I}$, then for each $I_d \le I_e$ we will have $p(I_d) \ge \hat{p}$. If in the interval $[I_{k-1}, I_k]$, where $I_{k-1} < I_k \le \bar{I}$, we have inequality $I_d(I_e) < I_e$, then

$$R(I_{k-1} < I_e < I_k) = \int_{I_{k-1}}^{I_k} tI_d(I_e) + p(I_d(I_e))[(t+f)(I_{cp}(I_e) - I_d(I_e)) - c)\alpha(I_e)dI_e$$

$$= \int_{I_{k-1}}^{I_k} tI_d(I_e) + p(I_d(I_e))\{(t+f)[(I_{cp}(I_e) - I_e) + (I_e - I_d(I_e))] - c\}\alpha(I_e)dI_e .$$

Then

$$\int_{I_{k-1}}^{I_k} tI_e + \frac{t}{t+f}[(t+f)(I_{cp}(I_e) - I_e) - c)\alpha(I_e)dI_e - R(I_{k-1} < I_e < I_k)$$

$$= \int_{I_{k-1}}^{I_k} [t(I_e - I_d(I_e))(1 - p(I_d(I_e)) + \Phi(I_e)(\hat{p} - p(I_d(I_e)))]\alpha(I_e)dI_e > 0$$

From the last term, it is concluded that $p^*(I_d(I_e)) = \hat{p} \quad \forall I_e \in [I_{k-1}, I_k]$. While in the interval $[I_{k-1}, I_k]$, where $I_{k-1} < I_k \le \bar{I}$, if we have $I_d(I_e) = I_e$, then

$$R(I_{k-1} < I_e < I_k) = \int_{I_{k-1}}^{I_k} tI_e + p(I_e)[(t+f)(I_{cp}(I_e) - I_e) - c)\alpha(I_e)dI_e = \int_{I_{k-1}}^{I_k} [tI_e + p(I_e)\Phi(I_e)]\alpha(I_e)dI_e .$$

Because $\Phi(I_e) \le 0 \;\; \forall I_e$, then we will have $p^*(I_d(I_e)) = \hat{p} \quad \forall I_e \in [I_{k-1}, I_k]$, where $I_{k-1} < I_k \le \bar{I}$.

Now we are going to analyze the case in which $I_e \ge \bar{I}$.

First we define $q_{(I_e)} = p_{(I_d(I_e))}$. From the former theorem we have $\frac{dG}{dI_e} \le q_{(I_e)}(t+f)\frac{dI_{cp}}{dI_e}$. This means that $G(I_{d(I_e)}, I_e, q_{(I_e)}) \le G(I_{d(\bar{I})}, \bar{I}, q_{(\bar{I})}) + \int_{\bar{I}}^{I_e} q_{(s)}(t+f)I'_{cp(s)}ds$.
Therefore, the government's net income for the interval is expressed as follows:

$$R(q_{(I_e)}) = \int_{\bar{I}}^{\infty} [G(I_{d(I_e)}, I_e, q_{(I_e)}) - q_{(I_e)}c]\alpha_{(I_e)}dI_e$$

$$\le \int_{\bar{I}}^{\infty} [(G(I_d(\bar{I}), \bar{I}, q(\bar{I})) + \int_{\bar{I}}^{\infty} q(s)(t+f)I'_{cp}(s)ds) - q(I_e)c]\alpha(I_e)dI_e$$
$$= G(I_d(\bar{I}), \bar{I}, q(\bar{I}))(1-\beta(\bar{I})) + \int_{\bar{I}}^{\infty}[\int_{\bar{I}}^{I_e} q(\bar{I})(t+f)I'_{cp}(s)ds) - cq(I_e)]\alpha(I_e)dI_e$$
$$= G(I_d(\bar{I}), \bar{I}, q(\bar{I}))(1-\beta(\bar{I})) + \int_{\bar{I}}^{\infty} q(I_e)[(t+f)I'_{cp}(I_e)(1-\beta(I_e)) - c\alpha(I_e)]dI_e$$

We define $H_{(I_e)} = (t+f)I'_{cp(I_e)}(1-\beta_{(I_e)}) - c\alpha_{(I_e)}$, in this case:

$$R(q_{(I_e)}) = G(I_{d(\bar{I})}, \bar{I}, q_{(\bar{I})})(1-\beta_{(\bar{I})}) + \int_{\bar{I}}^{\infty} q_{(I_e)}H_{(I_e)}dI_e \qquad (*)$$

with the help of " Sanchez & Sobel" proof [7] , the value of I will be found, which based on this value, the integral $\int_{\bar{I}}^{\infty} q(Ie)it(Ie)dIc$ will be maximum and optimum function $q^*_{(I_e)}$ will be in the following step function form $q^*_{(I_e)} = \begin{cases} \hat{p} & I_e < \hat{I}, \\ 0 & I_e \ge \hat{I} \end{cases}$ which makes the government income value maximum with respect to (*). With analysis of $p^*_{(I_d(I_e))} = q^*(I_e)$ case, the theorem is proved.

Now we consider the government income function as function of variable $\hat{I}$, and we will have:

$$R(\hat{I}) = \int_0^{\hat{I}} \{tI_e + \frac{t}{t+f}[(t+f)(I_{cp}(I_e) - I_e) - c]\}\alpha(I_e)dI_e + t\hat{I}(1-\beta(\hat{I})). \qquad (**)$$

If we consider risk function $\frac{1-\beta(I)}{\alpha(I)}$, (where $\beta_{(I)} = \alpha'_{(I)}$) the next theorem states under what conditions there is a unique point which maximizes the $R(\hat{I})$.

Theorem 3.4 *If risk function and* $I_{cp}{}_{(I)} - I$ *are non – increasing function and the function* $I_{cp}{}_{(I)} - I \to 0$ *when* $I \to \infty$. *The there is* $\hat{I}^* = \arg\max R(p^*(I_d(I_e)))$ *which satisfies the following condition* $[\,t(I_{cp}{}_{(\hat{I})} - \hat{I}\,) - c\hat{p}\,]\alpha_{(\hat{I})} + t(1-\beta_{(\hat{I})}) = 0$.

Proof: It is concluded from the statement (**) that:

$$\frac{dR(I)}{dI} = t[(I_{cp}{}_{(I)} - I) - c]\alpha_{(I)} + t(1-\beta_{(I)})$$

In this new statement, it is obvious that $1-\beta_{(I)} > 0$. If $t[(I_{cp}{}_{(I)} - I) - c] > 0, \ \forall I$, then $\frac{dR(I)}{dI} > 0 \ \forall I$. Since $\lim_{I\to\infty}(I_{cp}{}_{(I)} - I) = 0$, in this case, there is a value such as I_0, which the inequality $t[(I_{cp}{}_{(I)} - I) - c] < 0 \quad \forall I > I_0$ is held base on it. But $\frac{1-\beta_{(I)}}{\alpha_{(I)}}$ and $I_{cp}{}_{(I)} - I$ are non-increasing functions. Thus the derivation $\frac{dR(I)}{dI}$ is also the same and in addition $\frac{dR(0)}{dI} > 0$ and $\lim_{I\to\infty}\frac{dR}{dI} < 0$. As a result, there is a unique point which the government income function is maximum for it. To continue, we assume the function $\Phi(I_e)$ is defined as follows:

$$\Phi(I_e) > 0 \quad if \quad I_e < J, \qquad \Phi(I_e) \le 0 \quad if \quad I_e \ge J .$$

Theorem 3.5

1 – In the case of $I_e \ge J$, the government optimum strategy will be in the form of a step function. 2- In the case of $I_e < J$, the interval $[0, J]$ is divided into $0 = I_1 < I_2 < \ldots < I_k = J$ subintervals which for these subintervals the government optimum strategy is subsequently and continuously is in one of the following forms: either $I_d(I_e) = I_e$, and $p(I_d(I_e)) = 1$ or $I_d(I_e) < I_e$, and $p(I_d(I_e))$ is a step function which is defined as the following :

$$\exists \bar{I}_k \in [I_k, I_{k+1}]; \qquad p_{(I_d(I_e))} = \begin{cases} q_{1k} & if \quad I_e \in [I_k, \bar{I}_k) \\ q_{2k} & if \quad I_e \in [\bar{I}_k, I_{k+1}) \end{cases},$$

such that $q_{1k} \geq q_{2k} \geq \hat{p}$.

The relation between the values q_{1k}, q_{2k}, $\bar{I}_k$, I_k, and I_{k+1} are defined in the following mathematical relation :

$$G(I_{k+1}, \bar{I}_k, p(\cdot)) = G(I_{k+1}, I_{k+1}, p(\cdot)),$$
$$G(\bar{I}_k, I_k, p(\cdot)) = G(\bar{I}_k, \bar{I}_k, p(\cdot)).$$

Proof: Under condition $I_e \geq J$ the proof is similar to theorem 3. Thus me assume that $I_e < J$. First the interval $[0, J]$ is divided into subintervals $0 = I_1 < I_2 < \ldots < I_k = J$. Based on theorem 3, if we have $I_d(I_e) = I_e$ in the interval $[I_{k-1}, I_k]$, then government income in this interval is equal to:

$$R(I_{k-1} < I_e < I_k) = \int_{I_{k-1}}^{I_k} \{tI_e + p_{(I_e)}[(t+f)(I_{cp}(I_e) - I_e) - c]\}\alpha_{(I_e)} dI_e = \int_{I_{k-1}}^{I_k} [tI_e + p_{(I_e)}\Phi_{(I_e)}]\alpha_{(I_e)} dI_e$$

We note that the net government income in this interval is strictly increasing with respect to *p* and the value of *p* is not equal to one. According to theorem 3 in [7], in the preceding interval to $[I_{k-2}, I_{k-1}]$ which $I_{d(I_e)} < I_e$ is held, the government maximum income is obtained in the following strategy:

$$\exists \hat{I}: \qquad q_{(I_e)} = \begin{cases} q_{k-2} & if \quad I_e \in (I_{k-2}, \hat{I}) \\ q_{k-1} & if \quad I_e \in (\hat{I}, I_{k-1}) \end{cases},$$

which $q_{k-2} = q_{(I_{k-2})} \geq q_{k-1} = q_{(I_{k-1})}$.

In this case the government net income will be as follows:

$$\int_{I_{k-2}}^{I_{k-1}} [G(I_{d(I_e)}, I_e, p_{(I_e)}) - q_{(I_e)}c]\alpha_{(I_e)} dI_e = \int_{I_{k-2}}^{I_{k-1}} tI_{d(I_e)} + q_{(I_e)}[(t+f)(I_{cp}(I_e) - I_{d(I_e)}) - c]\alpha_{(I_e)} dI_e$$

If we consider $I_{d(I_e)}$ as a constant, then because $\Phi_{(I_e)} > 0$, the above income function is increasing with respect to $q_{(I_e)}$. The curve resulting from the below equilibrium condition

$$tI_{k-2} + q_{k-2}(I_{cp}(I_e) - I_{k-2})(t+f) = tI_e + P_{1(I_e)}(I_{cp}(I_e) - I_e)(t+f),.$$

When in there $P_{1(I_e)} = \dfrac{\hat{p}(I_{k-2} - I_e) + q_{(I_{k-2})}(I_{cp}(I_e) - I_{k-2})}{I_{cp}(I_e) - I_e}$.

Determines the auditing probability between $\lfloor I_{k-2}, \hat{I})$, in which $I_d = I_e$. (the probability which base on it, in the tax report I_e or I_{k-2}, the income for the taxpayer would be equal). Thus for the report I_{k-2} to be economical, the inequality condition $p_{(I_e)} < P_{1(I_e)} \neq 1$ should hold for him.

If we increase the amount of $q_{(I_{k-2})} = q_2$, then the government's achievements will grow. In a similar manner we can consider the same thing for the $(\hat{I}, I_k)$. Therefore when the q_{k-1} increases, the amount of government's achievements will increase. In the second case, the value of $P_2(I_{k-1})$ is defined based on $G(I_{k-1}, I_{k-1}, P(I_{k-1})) = G(\hat{I}, I_{k-1}, q_{k-1})$ and it is not equal to one.

4- Conclusions

In the analyses of the game theory model based on tax random error for optimum strategy of government auditory, is analyzed with this assumption to consider the error as a factor smaller then real income. the probable optimum auditing will be in the from of the step function which depends on real income and assessed income by tax payers and government optimum income is dependent on the specified parameters based on This strategy .

Refinances

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