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Optimal Fiscal Policy with Oil Revenues

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Abstract

This paper focuses on the impacts of oil revenues on government fiscal policy when we have externality of human capital in economic. Therefore, we devised a fiscal policy capable to make the decentralized economy to achieve the first-best equilibrium in the Uzawa-Lucas model. The results of this paper show that optimal policy requires making use of a subsidy to investment in human and physical capital. Human capital can be financed by oil revenues and tax on labor income and physical capital can be financed by oil revenues. Government size dependent to oil revenues: When share of oil revenue in GDP or ratio of oil revenue in physical capital increase, government size increases and conversely. The results show the return on the physical capital must be free of taxes, but tax on labor income needed to balance the government budget in the steady state or in the transitional phase.

Keywords: Optimal fiscal policy; Natural resources; Endogenous growth.

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1- Introduction

Countries with large share of government revenue comes selling a government owned exhaustible natural resources such as oil, most understand used oil revenue in currency consumption has negative effects on economic such as Dutch disease or voracity effect. On the other hand, large and unpredictable fluctuations in international oil price may make the determination of appropriate expenditure levels particularly difficult.

Studies show countries with great natural resource wealth tend nevertheless to grow more slowly than resource-poor countries the curse of natural resources (the observation that countries rich in natural resources tend to perform badly) has been shown empirically and analyzed in a number of recent studies. Such as Auty (1990), Gelb (1988), Sachs and Warner (1995, 1999, 2001), and Gylfason et al. (1999). Since so many poorer countries still have abundant natural resources, it is important to better understand the roots of failure in natural resource-led development (Sachs and Warner, p. 827).

Evidence that fiscal policy has been procyclical and has hence exacerbated the fluctuations in economic activity. In addition, a small reduction in oil prices could lead to very large financing needs in the near future. Finally, long-term fiscal sustainability positions in OPCs have worsened. (Villafuerte and Lopez-Murphy, 2010, p. 1)

When the government see to oil revenue as a consumption resource (in an economy that lacks a strong legal-political institutional infrastructures and is populated by multiple powerful groups) Powerful groups dynamically interact via a fiscal process that effectively allows open access to the oil revenue. In equilibrium, this leads to slow economic growth and a "voracity effect", by which a shock, such as a terms of trade windfall, perversely generates a more-than-proportionate increase in fiscal redistribution and reduces growth (Tornell AND Lane, 1999. P. 22).

If these countries see to the oil revenue as wealth and no income, then they must invest this revenue in human and physical capital.

The purpose of this paper is to devise a fiscal policy when governments have oil revenue and externality of human capital exist in economy. The government is one that taxes both physical and human capital income and oil revenues, then subsidizes investment in human capital. We assume that the government cannot resort to lump-sum taxes. The optimal fiscal policy requires the use of a time-varying subsidy rate to investment in human capital. Public spending can be financed by means of oil revenue and a time-varying tax rate on labor income, without the necessity of resorting to lump-sum taxation to balance the government budget either in the steady state or in the transitional phase. Physical capital income should be negative taxed or subsidy. Alternatively, the optimal growth path can be attained by means of a subsidy to human capital, which can be fully financed by oil revenue and constant tax on labor income. In this case, the optimal subsidy amounts to a constant share of output not only in the steady state but also in the transitory phase.

The remainder of this paper is organized as follows. Section 2.1. Describes the competitive equilibrium, section 2.2. The governmental planned economy with oil revenue, section 3 analyzes the optimal fiscal policy with oil revenue and section 4 concludes.

2- The model

2.1. Competitive equilibrium

The model used in this paper is the two-sectors model introduced by Uzawa (1965) and Lucas (1988), known as the Uzawa-Lucas model. In this model total output depends on physical and human capital and the saving rate endogenously determined by the preference and technology parameters. In particular, it is assumed that households maximize the discounted stream of utility arising from consumption.

The utility function used on this model is the constant elasticity of substitution, or isoelastic, function:

$$u(c) = \begin{Bmatrix} \dfrac{C^{1-\theta}}{1-\theta} & \text{for } \theta > 0\,, \theta \neq 1 \\ \ln c & \theta = 1 \end{Bmatrix} \quad (1)$$

In this model the elasticity of substitution between consumption at any two points in time, t and s, is constant and equal to $(1/\theta)$. When this instantaneous utility function is used to describe attitudes toward risk, θ has an alternative interpretation. It is then also the coefficient of relative risk aversion, defined as $-u''(c)c/\,u'(c)$. According to this definition, θ is the inverse of the elasticity of intertemporal substitution (Blanchard & Fischer, 1993 , p. 44).

The intertemporal utility derived by the agent is represented by function form (1):

$$Max\int_0^\infty e^{-\rho t}\frac{C^{1-\theta}}{1-\theta}dt \qquad \rho>0, \quad \theta>0 \tag{2}$$

Where ρ is the rate of time preference.

In function (3) H represents the stock of human capital. The coefficient u is the portion of human capital devoted to the production of output Y. Suppose that $0 \le u \le 1$, and let $1 - u$ be the portion of human capital devoted to the production of more human capital. This leads to an equation of motion for the human capital stock:

$$\dot{H} = \delta(1-u)H \tag{3}$$

To interpret δ, note that

$$\dot{H} = \gamma(1-u)H \quad \Rightarrow \quad \frac{\dot{H}}{H} = \gamma(1-u) \quad \Rightarrow \quad \gamma_H = \gamma(1-u) \tag{4}$$

Therefore:

$$u = 0 \Rightarrow \gamma_H = \gamma$$

The household's budget constraint is,

$$\dot{K} = (1-\tau_K)r_K + (1-\tau_H)wu\mathrm{H} - c + s_H w(1-u)\mathrm{H} \tag{5}$$

Where r is the rate of return on physical capital, w is the wage rate, τ_K is the rate of government taxes physical capital income, τ_H is the rate of labor income tax, and s_H is rate of subsidizes investment in education. The sole cost of education is foregone earnings, $w(1-u)$H, a fraction s_H of which is therefore financed by the government. In this equation, we ignore depreciation of physical capital.

The representative agent maximizes (2) subject to the constraints (3) and (5). For simplifying the subsequent exposition we shall slightly change the budget constraint (6), and express it equivalently as

$$\dot{K} = (1-\tau_K)rK + (1-\hat{\tau}_H)wu\mathrm{H} - c + s_H w(1-u)\mathrm{H} \tag{6}$$

Where

$$\hat{\tau}_K = \tau_K + s_H \tag{7}$$

We assume that the production function has a Cobb-Douglas form by constant returns to scale; here it takes a slightly different form:

$$Y = AK^{\alpha}(uH)^{1-\alpha} H_a^{\beta} \tag{8}$$

$$A > 0 , 0 < \alpha < 1 , \beta > 0$$

Where, K denotes the stock of physical capital and H represents the stock of human capital. The coefficient u is the portion of human capital devoted to the production of output Y.

In the market solution, the atomistic agents treat H_a as given. Suppose there is symmetry between the values of H_a and H. Because of the externality effects, the competitive solution differs from the planner's solution.

We shall assume that the government has a balanced-budget.So,

$$\tau_K rK + \tau_H wu\mathrm{H} = s_H w(1 - u)\mathrm{H} \tag{9}$$

Using (7), we can write it as follows:

$$\tau_K rK + \hat{\tau}_H wu\mathrm{H} = s_H w\mathrm{uH} \tag{10}$$

This equation is not included a consumption tax because it acts as a lump-sum tax in this framework. Let us denote the current value Hamiltonian of the household's utility maximization problem by L. L is as follows:

$$L = \frac{e^{-\rho t}(C^{1-\theta}-1)}{1-\theta} + \lambda_H e^{-\rho t}[\delta(1-u)H] + \lambda_K e^{-\rho t}\left[(1 - \tau_K)rK + (1 - \hat{\tau}_H)wu\mathrm{H} - c + s_H w(1 - u)\mathrm{H}\right] \tag{11}$$

Where λ_K and λ_H are the multipliers for the constraints (5) and (3), respectively.

The first order necessary conditions for an interior solution are

$$\frac{\partial L}{\partial C} = 0 , \frac{\partial L}{\partial u} = 0 , \frac{\partial(\lambda_H e^{-\rho t})}{\partial t} = -\frac{\partial L}{\partial H} , \frac{\partial(\lambda_K e^{-\rho t})}{\partial t} = -\frac{\partial L}{\partial K}$$

Hence, we have:

$$C^{-\theta} = \lambda_K \tag{12a}$$

$$\lambda_H \delta H = \lambda_K (1 - \hat{\tau}_H)w\mathrm{H} \tag{12b}$$

$$\dot{\lambda}_K = (\rho - (1 - \tau_K)r)\lambda_K \tag{12c}$$

$$\dot{\lambda}_H = (\rho - \delta(1-u))\lambda_H - ((1 - \hat{\tau}_H)w\mathrm{u} + s_H w)\lambda_K \tag{12d}$$

And transversality conditions is satisfied. Let $g_x = \frac{\dot{x}}{x}$ denote the growth rate of the variable *x*. the equilibrium condition H=H_a will be imposed, and the expressions for r and w will be taken into account. From first order conditions we obtain

$$g_c = ((1 - \tau_K)\mathrm{r} - \rho)/\theta \tag{13}$$

By profit maximiz, the labor and capital are used up to the point at which marginal product equates marginal cost: $= \alpha\, {}^{Y}\!/_{K}$, and $w = (1-\alpha)\, {}^{Y}\!/_{uH}$.

Then:

$$g_c = (\alpha(1 - \tau_K)\,{}^{\mathrm{Y}}\!/_{\mathrm{K}} - \rho)/\theta \tag{14}$$

Using the government budget constraint (10), Eq. (6) can be expressed as:

$$\dot{K} = rK + wuH - c \tag{15}$$

So:

$$g_K = {}^{Y}\!/_{K} - {}^{C}\!/_{K} \tag{16}$$

We have

$$\frac{\dot{\lambda}_K}{\lambda_K} - \frac{\dot{\hat{\tau}}_H}{1-\hat{\tau}_H} + \frac{\dot{w}}{w} = \frac{\dot{\lambda}_H}{\lambda_H} + \frac{\dot{\delta}}{\delta} \tag{17}$$

$$\frac{\dot{\lambda}_K}{\lambda_K} = (\rho - (1 - \tau_K)r) \tag{18}$$

$$\frac{\dot{u}}{u} = \frac{\dot{Y}}{Y} - \frac{\dot{w}}{w} - \frac{\dot{H}}{H} \tag{19}$$

$$\frac{\dot{\lambda}_H}{\lambda_H} = \rho - \delta(1 - \hat{\tau}_H + s_H)/(1 - \hat{\tau}_H) \tag{20}$$

By substituting the expression from (12b) into (12d) and Eq (17) to (20) we can obtain the growth rate of u as follows:

$$g_u = \frac{\delta(\alpha-\beta)u}{\alpha} + \frac{\tau_K Y}{K} - \frac{C}{K} + \frac{\delta s_H}{(1 - \hat{\tau}_H)\alpha} - \frac{\dot{\hat{\tau}}_H}{(1-\hat{\tau}_H)\alpha} + \frac{\delta(1-\alpha+\beta)}{\beta} \tag{21}$$

For a given policy path, the system (3), (14), (16) and (20) determines the dynamics of the decentralized economy. This system can be reformulated in terms of variables that are constant in the steady state, defining $M = KH^{(1-\alpha+\beta)/(\alpha-1)}$

and N=C/K. Then, we obtain

$$g_N = g_C - g_K$$

$$g_N = AM^{\alpha-1}u^{1-\alpha}((1-\tau_K)\alpha - \theta)/\theta - \rho/\theta + N \tag{22a}$$

$$g_u = \frac{\delta(\alpha-\beta)u}{\alpha} + \frac{\tau_K Y}{K} - \frac{C}{K} + \frac{\delta s_H}{(1 - \hat{\tau}_H)\alpha} - \frac{\dot{\hat{\tau}}_H}{(1-\hat{\tau}_H)\alpha} + \frac{\delta(1-\alpha+\beta)}{\beta} \tag{22b}$$

2-2-The Centrally Planned Economy

The central planner maximizes (1) subject to (2) and

$$\dot{K} = AK^{\alpha}H^{1-\alpha+\beta}u^{1-\alpha} - C + O \tag{23}$$

Where, O is oil revenue. Let L be the current value Hamiltonian of the planner's maximization problem,

$$L = \frac{e^{-\rho t}(C^{1-\theta}-1)}{1-\theta} + \lambda_H e^{-\rho t}[\delta(1-u)H] + \lambda_K e^{-\rho t}\left[AK^{\alpha}H^{1-\alpha+\beta}u^{1-\alpha} - C + O\right] \quad (24)$$

Then according to first order necessary conditions for an interior solution, we have:

$$C^{-\theta} = \lambda_K \quad (25a)$$
$$\lambda_H \delta H = \lambda_K (1 - \alpha) AK^{\alpha}H^{1-\alpha+\beta}u^{-\alpha} \quad (25b)$$
$$\dot{\lambda}_K = (\rho - A\alpha K^{\alpha-1}H^{1-\alpha+\beta}u^{1-\alpha})\lambda_K \quad (25c)$$
$$\dot{\lambda}_H = \big(\rho - \delta(1-u)\big)\lambda_H - ((1 - \alpha + \beta)AK^{\alpha}H^{-\alpha+\beta}u^{1-\alpha})\lambda_K \quad (25d)$$

And the usual transversality conditions are satisfied. If you compare results of the centrally planned economies with the decentralized economics, then we can obtain that there are two main qualitative differences between them. First, the tax rate on physical capital income influences the return to physical capital in the market economy but not the implicit interest rate used by the planner. Second, the productivity elasticity determined by the planner's accumulation of human capital is the social productivity of human capital, $1 - \alpha + \beta$, in companies with the private productivity, $1 - \alpha$. With using the result of first order condition, we have:

$$\frac{\dot{\lambda}_H}{\lambda_H} = \frac{\dot{\lambda}_K}{\lambda_K} + \beta\frac{\dot{K}}{K} - \beta\frac{\dot{u}}{u} + (\beta - \alpha)\frac{\dot{H}}{H} \quad (26)$$
$$\frac{\dot{\lambda}_K}{\lambda_K} = (\rho - A\alpha K^{\alpha-1}H^{1-\alpha+\beta}u^{1-\alpha}) \quad (27)$$
$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} + \frac{O}{K} \quad (28)$$
$$\frac{\dot{\lambda}_H}{\lambda_H} = \rho - \delta\big(1 - u\big) - \frac{\delta(1-\alpha+\beta)}{1-\alpha} \quad (29)$$

Then:

$$g_u = \frac{\delta(1-\alpha+\beta)u}{1-\alpha} - N + \frac{O}{K} + \frac{\delta(1-\alpha+\beta)}{\alpha} \quad (30)$$

and

$$g_N = (\alpha - \theta)AM^{\alpha-1}u^{1-\alpha}/\theta - \rho/\theta + N - O/K \quad (31)$$

If we denote $r = \alpha Y/K$, then

$$\frac{\dot{r}}{r} = \frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} \tag{32}$$

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{u}}{u} + (1-\alpha+\beta)\frac{\dot{H}}{H} \tag{33}$$

$$\frac{\dot{r}}{r} = -\frac{(1-\alpha)r}{\alpha} + \frac{\delta(1-\alpha+\beta)}{\alpha} + \frac{O}{K} \tag{34}$$

Also If we denote $O_k = O/K$, then

$$\frac{\dot{O_K}}{O_K} = \frac{\dot{O}}{O} - \frac{\dot{K}}{K} \tag{35}$$

$$\frac{\dot{O_K}}{O_K} = \frac{\dot{O}}{O} - \frac{r}{\alpha} + N - O_k \tag{36}$$

For given policy path, the system (29), (30), (33) and (35) determines the dynamic of centrally planned economy, then we have:

$$g_N = \frac{(\alpha-\theta)r}{\alpha\theta} - \frac{\rho}{\theta} + N - \frac{O}{K} \tag{37a}$$

$$g_u = \frac{\delta(1-\alpha+\beta)u}{1-\alpha} - N + \frac{O}{K} + \frac{\delta(1-\alpha+\beta)}{\alpha} \tag{37b}$$

$$g_r = -\frac{(1-\alpha)r}{\alpha} - (1-\alpha)\frac{O}{K} + \frac{\delta(1-\alpha+\beta)}{\alpha} \tag{37c}$$

$$g_{O_K} = g_O - \frac{r}{\alpha} + N - O_k \tag{37d}$$

This system is accessible to a phase diagram analysis similar to that performed by Barro and Sala-i-Martin (1995, Sec 5.2.2) and Arnold (2000) in the model without any externalities and has been used by Gomez (2003) with externalities.

when $g_r = 0$, then $r = \frac{\delta(1-\alpha+\beta)}{1-\alpha} - \alpha\frac{O}{K}$ is vertical and stable. When $g_N = 0$, we have $N = \frac{O}{K} + \frac{\rho}{\theta} - \frac{(\alpha-\theta)r}{\alpha\theta}$ that locus is increasing and unstable if $\alpha < \theta$, show In the left panel of Figure 2 is a phase diagram in the (*r, N*)-space. And that locus is decreasing and unstable if $\alpha > \theta$, that show In the right panel of Figure 2 is a phase diagram in the (*r, N*)- space.

Then, there is a unique and saddle point steady state (*r, N*). The top right panel of Figure 1 depicts a phase diagram in the (*r, N*)-space. Given that the economy is on its saddle path in the (*r, N*)-space, *N* converges monotonically.

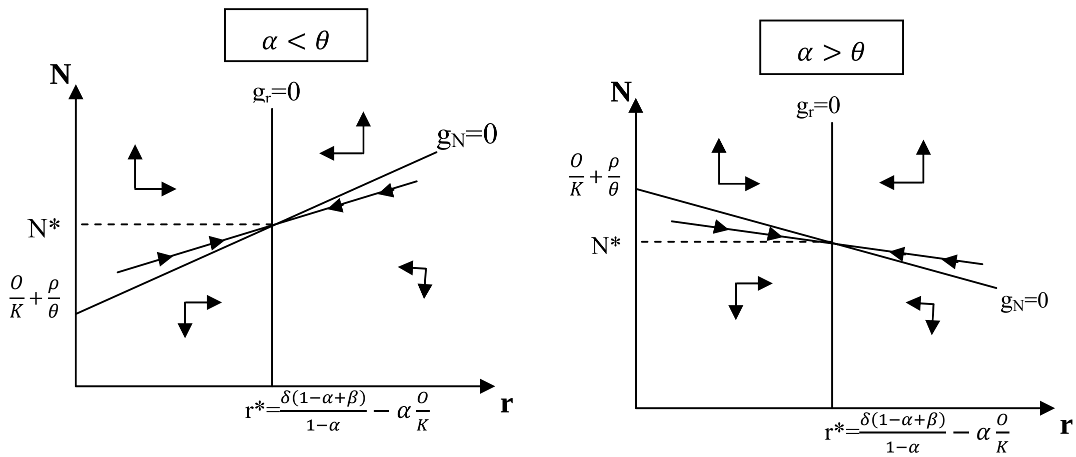

Figure 1: Phase Diagram in the (r, N)-Space

When $g_N = 0$, then $N = \frac{O}{K} + \frac{\rho}{\theta} - \frac{(\alpha-\theta)r}{\alpha\theta}$ is horizontal and stable in (*N, u*)- space. When $g_u = 0$, we have $N = \frac{O}{K} + \frac{\delta(1-\alpha+\beta)}{\alpha} + \frac{\delta(1-\alpha+\beta)}{1-\alpha}u$ that locus is increasing and unstable. Show In Figure 2 in the (*N,u*)- space. Then, there is a unique and saddle point steady state (*N,u*). The Figure 2 depicts a phase diagram in the (*N,u*)-space. Given that the economy is on its saddle path in the (*N,u*)-space, *N* converges monotonically.

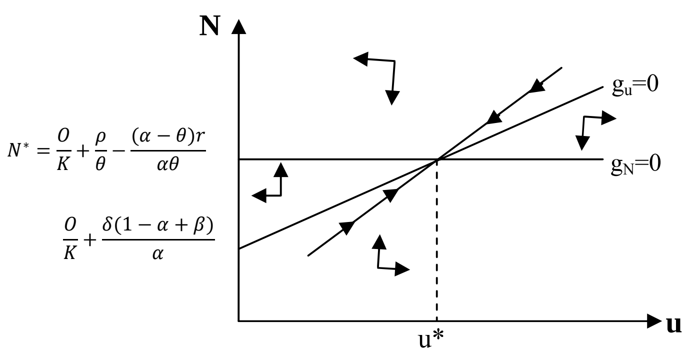

Figure 2: Phase Diagram in the (u, N) - Space

When $g_N = 0$, then $N = \frac{O}{K} + \frac{\rho}{\theta} - \frac{(\alpha-\theta)r}{\alpha\theta}$ and when $g_r = 0$, $\frac{O}{K} = \frac{\delta(1-\alpha+\beta)}{\alpha(1-\alpha)} - \frac{r}{\alpha}$ then $N = \frac{\delta(1-\alpha+\beta)}{\alpha(1-\alpha)} - \frac{r}{\theta} + \frac{\rho}{\theta}$ is horizontal and stable in (N, O_k)- space. When $g_{O_K} = 0$, we have $N = (\frac{r}{\alpha} - g_O) + O_k$ that locus is increasing and unstable if $\frac{r}{\alpha} > g_O$, show In the top panel of Figure 3 is a phase diagram in the (N, O_k)- space. And show In the down panel of Figure 3 is a phase diagram in the (N, O_k)- space if $\frac{r}{\alpha} < g_O$, that Then, there is a unique and saddle point steady state (N, O_k).

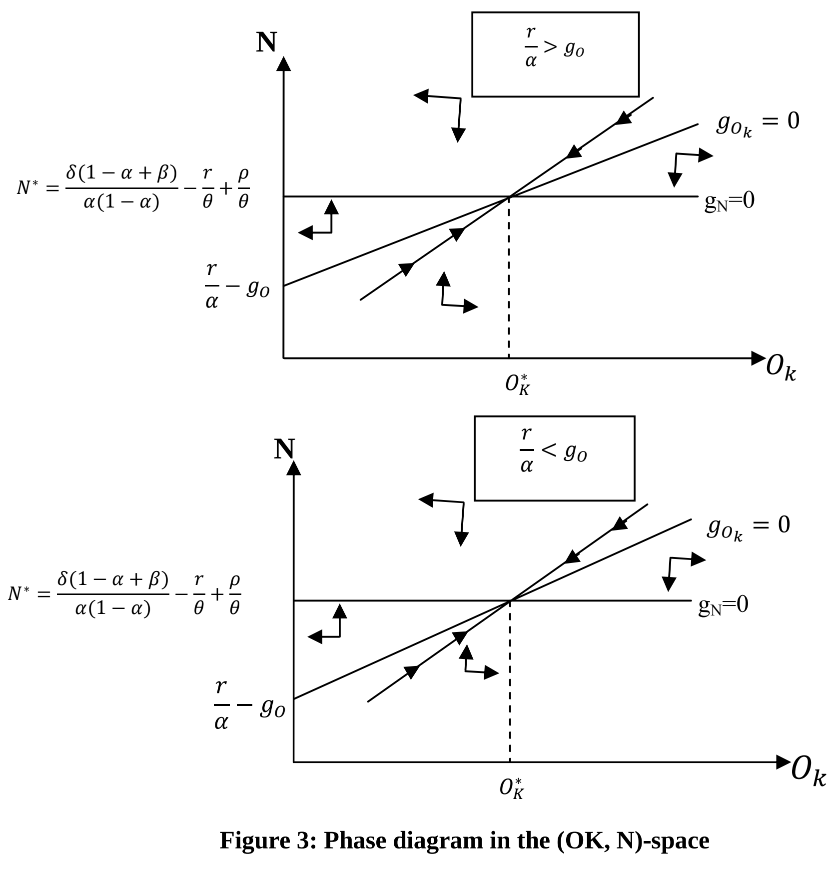

Figure 3: Phase diagram in the (OK, N)-space

Proposition 1: The steady state of the optimal-growth problem in the Uzawa-Lucas model when large share of government revenues comes selling a government owned exhaustible natural resources such as oil and average human capital has an external effect on productivity is a saddle-point.

The local stability analysis confirms the saddle-point property of the steady state. Linearizing the system (15) around the steady state (r∗, q∗,$O_K{}^*$, u∗) yields:

$$\begin{pmatrix} \dot{r} \\ \dot{N} \\ \dot{O_K} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} -\frac{(1-\alpha)}{\alpha} r^* & 0 & -(1-\alpha)r^* & 0 \\ \frac{(\alpha+\theta)}{\alpha\theta} N^* & N^* & -N^* & 0 \\ -\frac{1}{\alpha} O_K{}^* & O_K{}^* & -O_K{}^* & 0 \\ 0 & -u^* & u^* & \frac{\delta(1-\alpha+\beta)}{1-\alpha} u^* \end{pmatrix} * \begin{pmatrix} r - r^* \\ N - N^* \\ O_K - O_K{}^* \\ u - u^* \end{pmatrix}$$

There, we have the coefficient matrix that the characteristic roots are its diagonal elements. Two roots are positive and two is negative, which proves that the steady state is a saddle-point.

3- The optimal fiscal policy

Externalities are a case of market failure. When an externality exists, the prices in a market do not reflect the true marginal costs and/or marginal benefits associated with the goods and services traded in the market. A competitive economy will not achieve a Pareto optimum in the presence of externalities, because individuals acting in their own self interest will not have the correct incentives to maximize total surplus (i.e., the “invisible hand” of Adam Smith will not be “pushing folks in the right direction”). Because competitive markets are inefficient when externalities are present, governments often take policy action in an attempt to correct, or internalize, externalities (Zilberman, 1999, p. 1).

On the other hand in the absence of externalities, the competitive equilibrium is optimal and government intervention is not justified. But, optimal growth paths and competitive equilibrium paths do not coincide if externalities are present. In Lucas (1988) considers the case where average human capital has an external effect on the production of goods (Gomez, 2003, p. 917).

Human capital has positive externalities and one solution to develop the externalities are government subsidies on human capital. An efficient government intervention can provide the required incentives to compensate the market failure. Castrillo and Sanso (2000) and Gomez (2003) derive a fiscal policy that is capable to make the decentralized equilibrium with externalities be optimal in the Uzawa-Lucas model.

The key question to be addressed in this section is what fiscal policy is capable to make the decentralized economy replicate the first-best optimum attainable by a central planner and described by the mentioned system (37). First, Eq. (22a) together with Eq. (37a), leds us the decentralized economy will fully replicate the dynamic time path of *N* in the centrally planned economy if the tax rate on physical capital obtains to the following:

$$AM^{\alpha-1}u^{1-\alpha}((1-\tau_K)\alpha-\theta)/\theta-\rho/\theta+N=(\alpha-\theta)AM^{\alpha-1}u^{1-\alpha}/\theta-\rho/\theta+N-O/K$$

$$\tau_K=-\frac{\theta}{\alpha}\frac{O}{Y} \tag{38}$$

This refers that physical tax has negative relation with ratio of the oil revenue in GDP has negative effect on physical tax rate or government most pay subsides in physical capital.

$$s_K=\frac{\theta}{\alpha}\frac{O}{Y} \tag{39}$$

Also the above result shows while the ratio of oil revenue in GDP increases, consequently necessary physical capital subsides increase and vice versa.

In order to compute human capital subsides (s_H) we can equal the right hand sides of Eq. (22b) and (37b), after substituting $\tau_K=-\frac{\theta}{\alpha}\frac{O}{Y}$, yields the following relationship:

$$\frac{\delta(\alpha-\beta)u}{\alpha}+\frac{\tau_K Y}{K}-\frac{C}{K}+\frac{\delta s_H}{(1-\hat{\tau}_H)\alpha}-\frac{\dot{\hat{\tau}}_H}{(1-\hat{\tau}_H)\alpha}+\frac{\delta(1-\alpha+\beta)}{\beta}=\frac{\delta(1-\alpha+\beta)u}{1-\alpha}-N+\frac{O}{K}+\frac{\delta(1-\alpha+\beta)}{\alpha}$$

$$s_H=\left(1-\hat{\tau}_H\right)\left[\frac{\beta u}{1-\alpha}+\frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right]+\frac{\dot{\hat{\tau}}_H}{\delta} \tag{40}$$

This means that human capital subsides have positive effects on ratio of oil revenues in GDP. Also the result shows while ratio of oil revenue in GDP increase, necessary subsides of physical capital increases and vice versa.

If O=0, we can obtain derived subsides rate, are similar to derived rates by Gomez (2003), that all of them describes the dynamics of the market economy with externality and government intervention.

Because $\dot{\hat{\tau}}_H$ must be null in the steady state, optimal tax on labor income is constant in the steady state and transitional phase. Hence, Eq. (40) reduces to:

$$s_H = \left(1 - \hat{\tau}_H\right)\left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right] \tag{41}$$

$$\frac{\partial s_H}{\partial(\frac{O}{K})} = \left(1 - \hat{\tau}_H\right)\frac{(\alpha+\theta)}{\delta} \tag{42}$$

By solving (10) and substituting the optimal tax rate on physical capital income ($\tau_K = -\frac{\theta}{\alpha}\frac{O}{Y}$) we have:

$$\hat{\tau}_H = s_H + \frac{\theta}{1-\alpha}\frac{O}{Y} \tag{43}$$

Simultaneously solving the system (10) and (41), we can obtain the optimal quantity of $\hat{\tau}_H$,

$$\hat{\tau}_H = \frac{\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}+\frac{\theta}{1-\alpha}\frac{O}{Y}}{1+\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}}, \tag{44}$$

When

$$\frac{\theta}{1-\alpha}\frac{O}{Y} < 1 \Rightarrow \frac{O}{Y} < \frac{1-\alpha}{\theta} \Rightarrow 0 < \hat{\tau}_H < 1$$

Then $\frac{\partial s_H}{\partial(\frac{O}{K})} > 0$

Then the optimal subsidy rate, s_H, is

$$s_H = \left(1 - \frac{\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}+\frac{\theta}{1-\alpha}\frac{O}{Y}}{1+\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}}\right)\left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right] \tag{45}$$

If we return to the initial specification and impose a subsidy rate on human capital, Eqs. (7) and (44) yield tax rate on labor income as

$$s_H = \left(1 - \hat{\tau}_H\right)\left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right] \tag{46}$$

$$\tau_H = \frac{\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}+\frac{\theta}{1-\alpha}\frac{O}{Y}}{1+\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}} - \left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right]\left(1 - \frac{\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}+\frac{\theta}{1-\alpha}\frac{O}{Y}}{1+\frac{\beta u}{1-\alpha}+\frac{\alpha+\theta}{\delta}\frac{O}{K}}\right) \tag{47}$$

And we have:

$$0 < s_H < \frac{\left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right]}{1 + \left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right]} \Rightarrow 0 < \tau_H < 1$$

On the other hand government size, measured as the subsidy (or taxes) share of output, ϑ, is constant at any time, so:

$$\vartheta = \frac{G}{Y} = \frac{s_H + s_K}{Y} \tag{48}$$

Hence:

$$\vartheta = \frac{\left(1 - \frac{\theta}{1-\alpha}\frac{O}{Y}\right)\left(\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)O}{\delta\ \ K}\right)}{\left(1 + \frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)O}{\delta\ \ K}\right)Y} + \frac{\theta}{\alpha}\frac{O}{Y^2} \tag{49}$$

Finally we have:

$$\frac{\partial\vartheta}{\partial\left(\frac{O}{K}\right)} = \frac{(\alpha+\theta)\left(1 - \frac{\theta}{1-\alpha}\frac{O}{Y}\right)}{Y\delta\left(1 + \frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)O}{\delta\ \ K}\right)^2} > 0 \tag{50}$$

$$\frac{\partial\vartheta}{\partial\left(\frac{O}{Y}\right)} = \frac{(1 + \frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)O}{\delta\ \ K} - \alpha)\theta}{Y\alpha(1-\alpha)(1 + \frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)O}{\delta\ \ K})} > 0 \tag{51}$$

The following proposition summarizes the former findings.

Proposition 2: The Competitive equilibrium can attain the first-best equilibrium solution if investment in physical capital is taxed at rate $\tau_K = -\frac{\theta}{\alpha}\frac{O}{Y}$ or physical capital subsides equal to $s_K = \frac{\theta}{\alpha}\frac{O}{Y}$ and investment in human capital is subsidized at a rate $s_H = \left(1 - \frac{\frac{\beta u}{1-\alpha} + \frac{\alpha+\theta}{\delta}\frac{O}{K} + \frac{\theta}{1-\alpha}\frac{O}{Y}}{1 + \frac{\beta u}{1-\alpha} + \frac{\alpha+\theta}{\delta}\frac{O}{K}}\right)\left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right]$. physical capital subsides can be financed by oil revenue and human capital subsides can be financed by oil revenue and taxing human capital income at a rate $\tau_H = \frac{\frac{\beta u}{1-\alpha} + \frac{\alpha+\theta}{\delta}\frac{O}{K} + \frac{\theta}{1-\alpha}\frac{O}{Y}}{1 + \frac{\beta u}{1-\alpha} + \frac{\alpha+\theta}{\delta}\frac{O}{K}} - \left[\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right]\left(1 - \frac{\frac{\beta u}{1-\alpha} + \frac{\alpha+\theta}{\delta}\frac{O}{K} + \frac{\theta}{1-\alpha}\frac{O}{Y}}{1 + \frac{\beta u}{1-\alpha} + \frac{\alpha+\theta}{\delta}\frac{O}{K}}\right)$. Lump-sum taxation is not required to balance the government budget either in the steady state or in the transitory phase.

Proposition 2: Government size, measured as the subsidy (or taxes) share of output in this studies is ϑ. Measuring show Government size was affected to oil revenue:

$$\vartheta = \frac{\left(1 - \frac{\theta}{1-\alpha}\frac{O}{Y}\right)\left(\frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right)}{\left(1 + \frac{\beta u}{1-\alpha} + \frac{(\alpha+\theta)}{\delta}\frac{O}{K}\right)Y} + \frac{\theta}{\alpha}\frac{O}{Y^2}$$

When share of oil revenue in GDP or ratio of oil revenue in physical capital increase, government size increases and conversely.

4- Conclusion

In this paper we focus on the impact of oil revenue on government fiscal policy when we have externality of human capital in economic. Therefore we devised a fiscal policy capable to make the decentralized economy achieve the first-best equilibrium in the Uzawa-Lucas model. The optimal policy requires making use of a subsidy to investment in human and physical capital. Human capital can be financed by oil revenue and tax on labor income and physical capital can be financed by oil revenue. Government size is dependent to oil revenue, when share of oil revenue in GDP or ratio of oil revenue in physical capital increase, government size increases and conversely. The results show the return on the physical capital must be free of taxes, but tax on labor income needed to balance the government budget in the steady state or in the transitional phase.

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