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The Proposed Mathematical Models for Decision-Making and Forecasting on Euro-Yen in Foreign Exchange Market

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Abstract

In this paper two approaches for trading and forecasting on Euro-Yen exchange rates are suggested. In the first approach three decision-making models are developed to maximize profit of trades during a specific period. Traders have three options to perform a trade at each market time that are: (a) Opening a buy trade, (b) Opening a sell trade and (c) Refusal of trading. These options are considered in the models by using related decision variables. Results of these models conform to qualitative contents in literature of foreign exchange market and present trading strategy on the basis of the indicators to maximize profit. The aim of second approach is forecasting the direction of exchange rate (increase or decrease) over a specific period on the basis of values of indicators in previous time period. In this approach two heuristic models are developed to minimize mean of errors of forecasting. Then mean of errors of developed models are compared with four major classification algorithms. Results show that the proposed model has higher accuracy in forecasting

Keywords: Foreign Exchange Market, Forecasting, Classification Algorithms, Mean of Errors, Direction of Exchange Rate, Profit Maximization, EURJPY Exchange Rate.

1- Introduction

Initially an overview of foreign exchange market (Forex) is presented. In Forex market, the traders exchange different currencies via Internet. There are three major orders for trading.

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- **Buy Order:**

Consider that analysis expresses that the EURJPY exchange rate will increase soon. In this condition, the trader request "Buy" order.

- **Sell Order:**

Consider that analysis expresses that the EURJPY exchange rate will decrease soon. In this conditions, the trader request "Sell" order.

- **Close Order:**

Each opened trade must be closed at a time. The suitable time for trade closing depends to trader analysis. For example closing can be occurred when the trader is acquiescent for trade profit. As well as, closing can be occurred when the trader is afraid of the trade loss.

- **Volume:**

Volume is equal to amount of money that trader invest for trading in a time period.

There are four major exchange rates in each time period. A time period can be hourly, daily, weekly and etc.

- Exchange rate at the start of a period. This exchange rate is called Open.
- Maximum value of exchange rate during a period. This exchange rate is called High.
- Minimum value of exchange rate during a period. This exchange rate is called Low.
- Exchange rate at the end of a period. This exchange rate is called Close.

In this paper, the exchange rate at the opening time of a trade is called p_o and the exchange rate at the closing time of a trade is called p_c. If a buy trade is opened, the profit/loss of trade is equal to p_c-p_o-ct. ct is cost of each trade that belongs to broker and is equal for buy or sell trades. It is necessary to mention that ct depends to volume of trade. Also if a sell trade is opened, the profit/loss of trade is equal to p_o-p_c-ct. Moreover traders can assign two limits to their trades. These limits are called stop loss (SL) and take profit

(TP). After trade opening if the loss of that trade reaches to a specific limit (SL), the broker automatically close that trade. Also if the profit of a trade reaches to a specific limit (TP), the broker automatically close that trade. These limits are used for decreasing trade risks.

In Forex market, profit and loss of each trade is stated on the basis of the Pip. Pip is a criterion for exchange rate changes. For example consider that EURJPY exchange rate changes from 158.75 to 158.76. In this condition it is said that EURJPY exchange rate increases one pip. If EURUSD exchange rate changes from 1.4595 to 1.4594, it is said that EURUSD exchange rate decreases one pip. At whole one pip is equal to one unit change in the last digit of exchange rate.

There are two general approaches for analysis and forecasting in Forex market. The approaches are technical and fundamental. In technical methods, analysis and forecasting are performed on the basis of the exchange rate and technical indicators such as moving average, RSI and etc. In fundamental methods, analysis and forecasting are performed on the basis of the immense variables such as interest rate, unemployment rate, GDP and etc. On the basis of these investigations forecasting about increase or decrease of a currency versus other currencies is presented. Political events and immense decisions has affected on the fundamental analysis.

2- Literature Review

In this section a review about some of the previous researches about exchange rate forecasting and decision-making in foreign exchange market is presented. By the attention to paper approach, reviewed papers are about the technical approach.

J. Yao and Chew Lim Tan (2000) develop a neural network for forecasting in Forex market. Time series and moving average indicator are the inputs of back propagation neural network. They presented forecasting about Japanese yen, British pound, Switzerland Frank and Australian dollar versus US dollar. A. Muriel (2004) used of a chaotic moving model in physics. He tried to forecast trend of three major currencies versus US dollar. The model is tested on October 2003 date set. Fang-Mei Tseng et al. (2001) combined ARIMA (Auto Regressign Integrated Moving Average) in time seris and fuzzy regression model to forecasting NT (New Tawanian) dollar versus US dollar. The model enables decision makers and traders to forecast the worst and the best status in future. Chakradhara Panda and V.Narasimhan (2007) used of feed forward neural network to forecast Indian Rupee versus US dollar weekly exchange rate. Results show that proposed model is better than linear regression and random walk models. Vincent C.S Lee and Hsiao Tshung Wong (2007) used neural network to develop a risk management model. Aim of the model was forecasting direction and value of exchange rate changes. Proposed model was a combined model and used of ANN (Artificial neural network) and fuzzy logic tools.

Roberto Baviera (2002) investigated non reliable markovian behavior of exchange rate. He used from that to inference special rules for doing successful trades. Craig Ellis and Patrick Wilson (2005) developed an integrated approach for forecasting foreign exchange rate. They used of random walk framework to forecasting direction and value of exchange rates. They estimated different statistical criteria such as mean, standard deviation, p-value and etc. Pukthuanthong et al. (2007) examined random behavior hypothesis in foreign exchange market. The focus of this research is on the future contracts. They used of regression for detecting relations among two consecutive periods. Data set includes exchange rate of Canadian dollar, Australian dollar, Japanese yen, Switzerland frank, British pound and Euro versus US dollar. At the end a profitable strategy was presented. Gan et al. (1995) used multi layered perceptron (MLP) neural network to forecasting exchange rate of Switzerland frank, Deutsche Mark and Japanese yen versus US dollar. They examined two models with single and multivariate time series and compared them with random walk model. Oh, K.J. et al. (2005) by using of non linear programming and neural network developed a daily financial condition indicator (DFCI) for presenting time signals. Their Proposed indicator had capability to create a zone alarm region for forecasting stochastic financial crisis. At the end, he used of DFCI on Korean financial market.

Hau, H. et al. (2006) developed a balanced model among foreign exchange rate, price stock and capital flow. Their results show that net flow in foreign exchange market has a positive correlation with value of currency unit. Their Results show that forecasting on the daily, monthly and seasonal time horizons have acceptable confidence. Ahmad, S.M. et al. (2007) by combining the famous technical indicators, developed a fuzzy indicator for presenting buy and sell signals in Forex and stock market. Results of their research show that proposed indicator had higher confidence than other indicators. Mei-Chih-Chen et al. (2007) by using of moving average (MA), stochastic indicators (KD), moving average convergence divergence (MACD), relative strength index (RSI) and Williams %R (WMS %R) developed a dynamic stock portfolio decision-making assistance model to forecasting direction of price changes in Taiwan stock market. Thomas C. Shik. et al. (2007) compares RSI and moving average on six currencies. Terence Tai-Leung Chong and Wing-Kam Ng (2008) investigated the beneficiary efficacy of rules that extracted from RSI and MACD in London stock market.

3- Problem Definition

This paper includes two main approaches that include mathematical decision-making models. In first approach a decision-making model for trading in foreign exchange market is developed. There are many indicators for analysis and decision-making in Forex market. Traders usually use indicators for decision-making. RSI is an important indicator and is used in previous researches frequently. Some of those are mentioned in previous section. So in first approach RSI is used for decision-making about traders. Relative-Strength Index (RSI) is a useful and practical tool for showing exchange rates swing. RSI compares “Increased Quantity” of exchange rate or stock price by “Decreased Quantity” over a specific period and converts the quantities to a value between 0 and 100.

Aim of proposed model is specifying rules for trading to maximize the profit over specific time. Time interval is from beginning of 2002 to the end of 2007. Time periods are considered daily. It is considered that at the start of each day decision about trades are made. At the start of each day traders decide about opening and closing trades on the basis of RSI value. For this purpose three models are developed. Then all models are solved by Branch and Bound (B-B) method with Lingo 9.0 software.

In second approach a heuristic model for forecasting the direction of exchange rate changes are presented. Direction in each period has two states. Consider that exchange rate at the end of a day is greater than exchange rate at the beginning of that day. In this case it is said that exchange rate is increased and value of direction is equal to 1. Also consider that exchange rate at the end of a day is less than exchange rate at the beginning of that day. In this case it is said that exchange rate is decreased and value of direction is equal to 0. Direction of exchange rate changes is forecasted on the basis of EMA (exponential moving average) and RSI. EMA is weighted average exchange rate of previous days. EMA is a useful indicator that assigns grater weights to the latest data to respond faster to changes. In whole of this paper data of exchange rate of Euro versus Yen that is showed with EURJPY is used.

3-1- Models for Maximizing Profit

In this section three models for maximizing profit are presented. It is necessary to mention that the models are developed by the authors and don't adopted from other references.

3-1-1- Model (1)

In this case, it is considered that each opened trade is closed at the start of later day. Indeed the duration of each trade is one day. So traders have three options at start of each day:

- Opening a buy trade
- Opening a sell trade
- Refusal of trading

Traders decide on the basis of the RSI values at the start of each day. For this purpose RSI values are portioned to ten equal intervals. I_1 variable is defined for specifying the interval of RSI. Table 1 shows I_1 and RSI values.

Table 1: RSI Indicator and I1 Values

RSI	[0,10]	[10,20]	[20,30]	[30,40]	[40,50]	[50,60]	[60,70]	[70,80]	[80,90]	[90,100]
I_1	1	2	3	4	5	6	7	8	9	10

Parameters:

P_t: EURJPY exchange rate at the start of t-th day

CT: the cost of each trade

$I_{1,t}$: Value of I_1 variable at the start of t-th day

T: Set of days in considered period

J_{i_1} : Set of days that at the start of them, I_1 is equal to i_1 ($1\leq i_1\leq 10$)

Decision Variables:

Z_{0,i_1} : Suppose that I_1 is equal to i_1. If Z_{0,i_1} =1 it indicates don't performing trade and if Z_{0,i_1} =0 it indicates performing trade.

Z_{1,i_1} : Suppose that I_1 is equal to i_1. If Z_{1,i_1} =1 it indicates performing buy trade and if Z_{1,i_1} =0 it indicates don't performing buy trade.

Z_{2,i_1} : Suppose that I_1 is equal to i_1. If Z_{2,i_1} =1 it indicates performing sell trade and if Z_{2,i_1} =0 it indicates don't performing sell trade.

Objective Function:

$$\text{Max } Z= \sum_{i_1=1}^{i_1=10} \sum_{j\in J_{i_1}} (P_{j+1} - P_j - CT)Z_{1,i_1} + \sum_{i_1=1}^{i_1=10} \sum_{j\in J_{i_1}} (P_j - P_{j+1} - CT)Z_{2,i_1} \qquad (1\text{-}1)$$

Objective function is maximizing the profit of trades in considered period. First term of objective function expresses profit (loss) of buy trades and second term expresses profit (loss) of sell trades. Each of two terms includes cost of trades.

Constraint:

$$Z_{0,i_1} + Z_{1,i_1} + Z_{2,i_1} = 1 \qquad \forall 1 \leq i_1 \leq 10 \qquad (1\text{-}2)$$

Constraint (1-2) expresses that in each day, only one of the buy, sell or refusal is given.

3-1-2- Model (2)

In model (1) duration of each trade was one day. But in model (2) duration of each trade can be more than one day. Indeed in this model there is not any constraint about duration of trades. In this model parameters are defined similar to model (1). In follow decision variables of model (2) are defined.

Decision Variables:

Theses variable are binary and indicates doing an action (1) or not doing an action (0).

$Z_{0,1,i_1}$: Open a buy trade (when there isn't an opened trade) when I_1 is equal to i_1

$Z_{0,2,i_1}$: Open a sell trade (when there isn't an opened trade) when I_1 is equal to i_1

$Z_{0,3,i_1}$: Refusal to trading (when there isn't an opened trade) when I_1 is equal to i_1

$Z_{1,1,i_1}$: Hold an opened buy trade when I_1 is equal to i_1

$Z_{1,2,i_1}$: Close an opened buy trade when I_1 is equal to i_1

$Z_{1,3,i_1}$: Close an opened buy trade and opening a sell trade when I_1 is equal to i_1

$Z_{2,1,i_1}$: Hold an opened sell trade when I_1 is equal to i_1

$Z_{2,2,i_1}$: Close an opened sell trade when I_1 is equal to i_1

$Z_{2,3,i_1}$: Close an opened sell trade and opening a buy trade when I_1 is equal to i_1

$N_{0,t}$: If there isn't any opened trade $N_{0,t}$ is equal to 1 and otherwise is equal to 0.

$N_{1,t}$: If there is an opened buy trade $N_{1,t}$ is equal to 1 and otherwise is equal to 0.

$N_{2,t}$: If there is an opened sell trade $N_{1,t}$ is equal to 1 and otherwise is equal to 0.

Objective Function:

$$MaxZ = \sum_{i_1=1}^{i_1=10} \sum_{j \in J_{i_1}} [N_{0,j}(-CT(Z_{0,1,i_1} + Z_{0,2,i_1}))] + \sum_{i_1=1}^{i_1=10} \sum_{j \in J_{i_1}} [N_{1,j}((P_{j+1} - P_j)(Z_{1,1,i_1}) - CT(Z_{1,3,i_1}))]$$

$$+ \sum_{i_1=1}^{i_1=10} \sum_{j \in J_{i_1}} [N_{2,j}((P_j - P_{j+1})(Z_{2,1,i_1}) - CT(Z_{2,3,i_1}))]$$

(2-1)

Objective function indicates the profit/loss of trades in whole of period. Depending on that each of the $N_{0,j}$، $N_{1,j}$، $N_{2,j}$ variables is equal to 1, the respective term is activated. Each term includes profit/loss of trades and cost of trade opening. First term calculates cost of trade opening when there isn't any opened trade. Second term calculates profit/loss of opened buy trades and cost of sell trade opening. Third term calculates profit/loss of opened sell trades and cost of buy trade opening.

Constraints:

$$Z_{0,1,i_1} + Z_{0,2,i_1} + Z_{0,3,i_1} = 1 \qquad \forall t \varepsilon J_{i_1} \qquad 1 \le i_1 \le 10 \qquad (2\text{-}2)$$

$$Z_{1,1,i_1} + Z_{1,2,i_1} + Z_{1,3,i_1} = 1 \qquad \forall t \varepsilon J_{i_1} \qquad 1 \le i_1 \le 10 \qquad (2\text{-}3)$$

$$Z_{2,1,i_1} + Z_{2,2,i_1} + Z_{2,3,i_1} = 1 \qquad \forall t \varepsilon J_{i_1} \qquad 1 \le i_1 \le 10 \qquad (2\text{-}4)$$

$$N_{0,t} + N_{1,t} + N_{2,t} = 1 \qquad 1 \le t_1 \le T \qquad (2\text{-}5)$$

$$N_{1,t+1} = N_{1,t} - N_{1,t}(Z_{1,2,t,i_1} + Z_{1,3,t,i_1}) + N_{0,t}(Z_{0,1,t,i_1}) \quad \forall t \varepsilon J_{i_1} \quad 1 \le i_1 \le 10 \quad (2\text{-}6)$$

$$N_{2,t+1} = N_{2,t} - N_{2,t}(Z_{2,1,t,i_1} + Z_{2,2,t,i_1}) + N_{0,t}(Z_{0,2,t,i_1}) \quad \forall t \varepsilon J_{i_1} \quad 1 \le i_1 \le 10 \quad (2\text{-}7)$$

Constraint (2-2) expresses that if tehre is not any opened trade, only one of the buy, sell or refusal decisions is made.

Constraint (2-3) expresses that if there is an opened buy trade, only one of the hold, close or close buy trade and open sell trade decisons is made.

Constraint (2-4) expresses that if tehre an opened sell trade, only one of the hold, close or close sell trade and open sbuy trade decision is made.

Constraint (2-5) expresses that in each day there is at most one buy or sell opened trade. This constraint is logical. For example consider a buy trade is opened and at the start of a later day trader feels that the exchange rate will decrease. In this case he can open a sell trade. So it isn't logical that the buy trade is remained in opened status. So trader should close buy trade and open a sell trade. Similar description can be presented about condition that a sell trade is opened.

Constraints (2-7) and (2-7) update $N_{1,t+1}$ *and* $N_{2,t+1}$ values on the basis of the $N_{1,t}$ *and* $N_{2,t}$ values and decision that was made in previous day. It is obvious that $N_{0,t}$ value is attained from constraint (4) for each day.

3-1-3- Model (3)

In this procedure an additional aspect of trades is considered. In foreign exchange market, traders can assign two limits to their trades. These limits are called stop loss (SL) and take profit (TP). After trade opening if the loss of that trade reaches to a specific limit (SL), the broker automatically close that trade. As well as if the profit of a trade reaches to a specific limit (TP), the broker automatically close that trade. In this section, trade limits (SL and TP) is considered in proposed model and a new model on the basis of trade limits is developed. In model (3) three additional parameters are added.

Parameters:

TP: Take profit limit

SL: Stop loss limit

Note that values of TP and SL will express on the basis of Pip.

Decision Variables:

L_t: Amount of profit/loss of a trade at the start of t-th day

$V_t, W_t, X_t, A_t, B_t, F_t$: Binary variables that is used in constraints

Other decision variables are similar to model (2).

Objective function in model (3) is similar equal to model (2). Constraints of model (3) are as follows:

Constraints:

$$Z_{0,1,i_1} + Z_{0,2,i_1} + Z_{0,3,i_1} = 1 \quad \forall t \varepsilon J_{i_1} \quad 1 \le i_1 \le 10 \tag{3-1}$$

$$Z_{1,1,i_1} + Z_{1,2,i_1} + Z_{1,3,i_1} = 1 \quad \forall t \varepsilon J_{i_1} \quad 1 \le i_1 \le 10 \tag{3-2}$$

$$Z_{2,1,i_1} + Z_{2,2,i_1} + Z_{2,3,i_1} = 1 \quad \forall t \varepsilon J_{i_1} \quad 1 \le i_1 \le 10 \tag{3-3}$$

$$N_{0,t} + N_{1,t} + N_{2,t} = 1 \quad 1 \le t \le T \tag{3-4}$$

$$L_{t+1} = (1 - N_{0,t+1})L_t + (N_{1,t+1})(P_{t+1} - P_t) + (N_{2,t+1})(P_t - P_{t+1}) \tag{3-5}$$

$$L_t \le SL + M * V_t \tag{3-6}$$

$$L_t + M * W_t \ge TP \tag{3-7}$$

$$F_t + M * X_t \ge 1 \tag{3-8}$$

$$V_t + W_t \le 1 + M * A_t \tag{3-9}$$

$$X_t \le 0 + M * B_t \tag{3-10}$$

$$A_t \ge B_t \tag{3-11}$$

$$N_{1,t+1} = (1 - F_{t+1}) * (N_{1,t} - N_{1,t}(Z_{1,2,i_1} + Z_{1,3,i_1}) + N_{0,t}(Z_{0,1,i_1})) \quad \forall t \varepsilon J_{i_1} \quad 1 \le i_1 \le 10 \tag{3-12}$$

$$N_{2,t+1} = (1 - F_{t+1}) * (N_{2,t} - N_{2,t}(Z_{2,1,i_1} + Z_{2,2,i_1}) + N_{0,t}(Z_{0,2,i_1})) \quad \forall t \varepsilon J_{i_1} \quad 1 \le i_1 \le 10 \tag{3-13}$$

Constraints (3-1) to (3-4) are similar to constrains of model (2). Constraint (3-5) expresses that the profit/loss of a trade equals to profit/loss of the trade at the start of the previous day plus to differentiation between exchange rate at the start of (t+1)-th day and t-th day. L_{t+1} depends to the L_t and the trade that are is opened at the start of (t+1)-th day as follows:

- If a buy trade at the start of day (t+1) is opened, then the $N_{1,t+1}$ is equal to 1. So on the basis of the constraint (3-4) $N_{0,t+1}$ and $N_{2,t+1}$ are equal to 0. Therefore on constraint (3-7) states that $L_{t+1} = L_t + (P_{t+1} - P_t)$.

- If a sell trade at the start of (t+1)-th day is opened, then the $N_{2,t+1}$ is equal to 1. So on the basis of the constraint (3-4) $N_{0,t+1}$ and $N_{1,t+1}$ are equal to 0. Therefore constraint (3-7) states that $L_{t+1} = L_t + (P_t - P_{t+1})$
- If there isn't any opened trade at the start of (t+1)-th day, then the $N_{0,t+1}$ is equal to 1. So on the basis of the constraint (3-4) $N_{1,t+1}$ and $N_{2,t+1}$ are equal to 0. Therefore constraint (3-7) states that $L_{t+1} = 0$

The above descriptions show that the constraint (3-5) calculates the profit/loss of each trade correctly.

Constraints (3-6) to (3-11) ensure that if L_t is equal or less than SL or L_t is equal or greater than TP, the opened trade at the start of day t is closed. If $L_t \leq SL$ then constraint (3-6) gives that $V_t = 0$. Also if $L_t \geq TP$ then constraint (3-7) gives that $W_t = 0$. If $L_t \leq SL$ or $L_t \geq TP$ then $V_t + W_t \leq 1$. In this case constraint (3-9) gives that $A_t = 0$. Then Constraint (11) gives that $B_t = 0$. After that constraint (3-10) gives that $X_t = 0$. Then constraint (3-8) gives that $F_t \geq 1$. Note that F_t is a binary variable. So $F_t = 1$. Constraints (3-12) and (3-13) express that if the F_t is equal to 1 then the opened trade is closed. Because if $F_t = 1$ then $(1-F_t) = 0$. So $N_{1,t}$ and $N_{2,t}$ are equal to 0. So at the beginning of t-th day, $N_{0,t}$ is equal to 1. So at the beginning of (t+1)-th day there isn't any opened trade.

3-2- Models for Minimizing Mean of Forecasting Errors

In previous section input data was RSI value at the start of each day and objective function was profit. Three models were developed for that purpose. In this section two models are developed for forecasting about direction of exchange rate in a day. In this section models with below specifications are developed.

- Objective function is minimizing error of forecasting.
- RSI and another useful indicator are considered.
- Indicator values for some previous days are considered.

In this section two indicators are used for forecasting direction in each day. These indicators are exponential moving average (EMA) and RSI.

Goal of this section is developing a heuristic method for daily direction forecasting on the basis of values of number of indicators in previous days. The considered indicators are EMA and RSI. For each day, values of indicators at the start of that day and 15 previous days are considered. Data set of this section includes EURJPY daily exchange rate in 2006, 2007 and 2008 years. Data set is divided in to two sections. First section that includes 2006, 2007 and first six months of 2008 is used for training. Next section that includes second six months of 2008 is used for model testing.

3-2-1- Model (4)

For direction forecasting a heuristic approach is offered. First a linear expression (function) of indicators is defined. Then on the basis of the value of function, direction of exchange rate in future day is forecasted. If value of the function is less than or equal to a specific value (That is called L), forecasted value of direction is 0 (0 indicates that exchange rate in that day will decrease). Also if value of the function is greater than L, forecasted value of direction is 1 (1 indicates that exchange rate in that day will increase).

Parameters:

EMA_{t-i}: Value of exponential moving average at the start of (t-i)-th day ($0 \leq I \leq 15$)

RSI_{t-i}: Value of RSI at the start of (t-i)-th day ($0 \leq I \leq 15$)

D_t: Direction of exchange rate in t-th day. 0 indicates that exchange rate at t-th day is decreased. 1 indicates that exchange rate at t-th day is increased.

L: Constants value that specifies forecasted direction on the basis of the F_t

T: Number of days that are used for training (In data set that is selected, T is equal to 630)

Decision variables:

F_t: linear function of indicators

PD_t: Forecasted direction of daily exchange rate in t-th day.
A_{t-i}: Coefficient of EMA_{t-i} in F_t
B_{t-i}: Coefficient of RSI_{t-i} in F_t
y_t, z_t: Binary variables that are used in constraints
H: A variable that is used in constraints

Objective Function:

$$Min\ Z = (\sum_{t=1}^{t=T} \left| PD_t - D_t \right|)/T \tag{4-1}$$

The goal is minimizing error of forecasting. If forecasted direction is equal to actual direction, $\left| PD_t - D_t \right|$ is equal to 0. Also if forecasted direction isn't equal to actual direction, $\left| PD_t - D_t \right|$ is equal to 1. Dividing the whole expression by T is resulted in attaining mean of errors. For objective function linearization a variable that is called H is defined and two constraints are added to set of constraints. So the objective function is defined as below:

$$Min\ Z = H \tag{4-2}$$

Constraints:

$$F_t = (\sum_{i=0}^{i=15} A_{t-i} EMA_{t-i} + \sum_{i=0}^{i=15} B_{t-i} RSI_{t-i})/32 \qquad 1 \le t \le T \tag{4-3}$$

$$F_t \le L + My_t \qquad 1 \le t \le T \tag{4-4}$$

$$F_t + Mz_t \ge L \qquad 1 \le t \le T \tag{4-5}$$

$$PD_t = y_t \qquad 1 \le t \le T \tag{4-6}$$

$$y_t + z_t = 1 \qquad 1 \le t \le T \tag{4-7}$$

$$H \ge (\sum_{t=1}^{t=T} (PD_t - D_t))/T \qquad 1 \le t \le T \tag{4-8}$$

$$H \ge (\sum_{t=1}^{t=T} (D_t - PD_t))/T \qquad 1 \le t \le T \tag{4-9}$$

Constraint (4-3) calculates linear function F_t on the basis of the EMA and RSI at start of t-th day and fifteen previous days. F_t uses 32 expressions. So the whole expression is divided on 32. Constraint (4-4) and (4-5) Compare F_t and L. If F_t is less than or equal to L, y_t is equal to 0. So on the basis of the considered approach forecasted value of direction is must be equal to 0. Constraint (4-6) sets forecasted value of direction equal to 0. Also if F_t is greater than L, z_t is equal to 0. So on the basis of the considered approach forecasted value of direction must be equal to 1. Constraint (4-7) results in y_t is equal to 1. Constraint (4-6) sets forecasted value of direction equal to 1. Constraints (4-8) and (4-9) result in that H variable is equal to $(\sum_{t=1}^{t=T} |PD_t - D_t|)/T$.

3-2-2- Model (5)

Model (4) is solved by branch and bound approach with Lingo 9.0 software. Optimal solution results in that objective function is equal to 0. That indicates that the model specifies A_t and B_t coefficients to adjust F_t with L and direction of exchange rates. Indeed model specifies A_t and B_t so that in each day with direction 0, F_t is less than or equal to L. Also A_t and B_t is specified so that in each day with direction 1, F_t is greater than L. Unfortunately results of applying coefficients on testing data set are not acceptable. Mean of errors on testing data set more than 50%. This condition is similar to a usual problem that can occur during training and testing a neural network. The problem is memorizing. Memorizing occurs when a neural network considers whole relations between inputs and outputs and tries to cover all detail of relations. So the built network doesn't have generality property to adjust with new data set. In this condition accuracy of model on training data set is much greater than accuracy of model on testing data set (Daniel T. Larose (2005)). For reducing this problem a heuristic approach can be applied. Training data set is divided to two sections. A constraint is added to model to ensure that difference between mean of errors of two sections is not greater than a specific limit. Indeed in this approach, values of A_{t-i} and B_{t-i} are attained to minimizing error of forecasting on first section of training data set.

The other problem against generality that can occurs is model biasing to forecasting one value much more than the other values. For example in our case, a model maybe biased to forecast most of directions equal to 0 or equal to 1. A constraint is added to model against this problem.

Parameters:

Most of parameters are similar to model (4). Some new parameters are defined as below:

T_1: Number of days of first section of training data set. (T_1 is selected equal to 580)

Gap1: Upper limit of difference between mean of errors of two section of training set

Gap2: Upper limit of difference between mean of errors of days of second section of training set that predicted direction is equal to 0 and days of second section of training set that predicted direction is equal to 1

Decision Variables:

Decision variables are similar to model (4). Some new constraints are defined as below:

H_1, H_2, H_3: Three variables that are used in constraints

Objective Function:

$$Min\ Z = \left(\sum_{t=1}^{t=T_1} \left|PD_t - D_t\right|\right) / T_1 \qquad (5\text{-}1)$$

Objective function is minimizing mean of errors of first section of training data set.

Constraints:

Most of constraints are similar to model (4).

Also two below constraints are added to model.

$$\left| \left(\left(\sum_{t=1}^{t=T_1} \left|PD_t - D_t\right| \right) / T_1 \right) - \left(\left(\sum_{t=T_1}^{t=T} \left|PD_t - D_t\right| \right) / (T - T_1) \right) \right| \le Gap1 \qquad (5\text{-}2)$$

$$\left| \sum_{t=T_1}^{T} y_t - \sum_{t=T_1}^{T} z_t \right| \le Gap2 \tag{5-3}$$

Constraint (5-2) ensures that difference between mean of errors of two sections of training data set is not greater than a specific limit (That is called Gap1). Also constraint (5-3) ensures that difference among number of days that forecasted value is equal to 0 and number of days that forecasted value is equal to 1, is not greater than a specific limit (That is called Gap2).

For objective and constrains linearization three variables that are called H_1, H_2 and H_3 are defined. Then objective function and constraints are changed as below:

Objective Function:

$$Min\ Z = H_1 / T_1 \tag{5-4}$$

Constraints:

$$H_1 \ge (\sum_{t=1}^{t=T} (PD_t - D_t)) \qquad 1 \le t \le T \tag{5-5}$$

$$H_1 \ge (\sum_{t=1}^{t=T} (D_t - PD_t)) \qquad 1 \le t \le T \tag{5-6}$$

$$H_2 \ge (\sum_{t=T_1}^{T} y_t - \sum_{t=T_1}^{T} z_t) \tag{5-7}$$

$$H_2 \ge (\sum_{t=T_1}^{T} z_t - \sum_{t=T_1}^{T} y_t) \tag{5-8}$$

$$H_3 \ge (H_1 / T_1) - (H_1 / (T - T_1)) \tag{5-9}$$

$$H_3 \ge (H_1 / (T - T_1)) - (H_1 / T_1) \tag{5-10}$$

$$H_3 \le Gap1 \tag{5-11}$$

$$H_2 \le Gap2 \tag{5-12}$$

It is assumed that Gap1 is equal to 0.1 (10%). Because 10% difference among accuracy on training data set and testing data set is acceptable.

Note that it is known that in average number of days with direction 0 and number of days with direction 1 is approximately equal. So it is logical that number of days that forecasting is stated that exchange rate will increase is equal to number of days that forecasting is stated that exchange rate will decrease. So Gap2 is should be selected as low as possible. But for values lower than 0.46 the model is infeasible. So Gap2 is set equal to 0.46. Average value of EMA and RSI is equal approximately to 105. So L is defined equal to numbers that are near to 105. Model is run for L=80, 90, 100, 110, 120. Initially model is run on training data set. Then attained values of A_{t-i} and B_{t-i} are used on testing data set.

4- Computational Results and Discussion

In this section results of solving models are presented.

4-1- Results of Model (1)

Model (1) is solved. Global optimal solution is found. The objective value is 46.7. Values of decision variables are show in table 2.

Table 2: Decision Variable Values in Global Solution for Model (1)

Decision variable	Value	Decision variable	Value	Decision variable	Value
$Z_{0,1}$	0	$Z_{1,1}$	0	$Z_{2,1}$	1
$Z_{0,2}$	0	$Z_{1,2}$	0	$Z_{2,2}$	1
$Z_{0,3}$	0	$Z_{1,3}$	1	$Z_{2,3}$	0
$Z_{0,4}$	0	$Z_{1,4}$	1	$Z_{2,4}$	0
$Z_{0,5}$	0	$Z_{1,5}$	1	$Z_{2,5}$	0
$Z_{0,6}$	0	$Z_{1,6}$	1	$Z_{2,6}$	0
$Z_{0,7}$	1	$Z_{1,7}$	0	$Z_{2,7}$	0
$Z_{0,8}$	0	$Z_{1,8}$	0	$Z_{2,8}$	1
$Z_{0,9}$	0	$Z_{1,9}$	0	$Z_{2,9}$	1
$Z_{0,10}$	0	$Z_{1,10}$	0	$Z_{2,10}$	1

Note that the maximum value for RSI indicator in data set is equal to 9 and the minimum value is equal to 3. So values of $Z_{0,1}$, $Z_{1,1}$, $Z_{2,1}$, $Z_{0,2}$, $Z_{1,2}$, $Z_{2,2}$, $Z_{0,10}$, $Z_{1,10}$ and $Z_{2,10}$ doesn't affect objective function value. So discussion and analysis is performed on other decision variables.

In literature of Forex market much analysis is expressed. Robert D. Edwards and John Magee in (2001) stated that RSI value lower than 30 is an indication to that exchange rate places on bottom and is a signal for exchange rate increase. So in this case a good opportunity for buy trade is occurred. Also they stated that RSI values higher than 70 is an indication to that exchange rate places on top and is a signal for exchange rate decrease. So in this case a good opportunity for sell trade is occurred. For investigate two mentioned hypotheses RSI values in [0, 30] and [70,100] intervals are considered. Note that RSI values higher than 70 is equivalent to that $I_{1,t}$ is equal to 8 or 9 or 10. Also RSI values lower than 30 is equivalent to that $I_{1,t}$ is equal to 1 or 2 or 3. So an analysis is performed on decision variables that their I_1 values are equal to 1, 2, 3, 4, 5 or 6. On the basis of data in Table 2, these results are attained:

- $Z_{1,3}$ is equal to 1. This states that when RSI is in [20, 30], opening a buy trade is an optimal decision.
- $Z_{2,8}$ is equal to 1. This states that when RSI is in [70, 80], opening a sell trade is an optimal decision.
- $Z_{2,9}$ is equal to 1. This states that when RSI is in [80, 90], opening a sell trade is an optimal decision.

These results are equivalent to analysis that is said by Robert D.Edwards, John Magee. (2001). In other words, results of model confirm analysis that is said in literature about RSI.

4-2- Results of Model (2)

Model (2) is solved. Local optimal solution is found. The value of objective function is equal to 82.31. It is notable that upper objective bound of objective function (profit) is attained equal to 82.31. So the local optimal solution is a global solution for model (2). Optimal value of objective function for model (2) is much greater than of objective function for model (1). It shows that constraint relaxation about trade duration results in increase profit of trades. Values of decision variables are show in table 3.

Table 3: Decision Variable Values in Global Solution for Model (2)

Decision Variable	$Z_{0,1,1}$	$Z_{0,1,2}$	$Z_{0,1,3}$	$Z_{0,1,4}$	$Z_{0,1,5}$	$Z_{0,1,6}$	$Z_{0,1,7}$	$Z_{0,1,8}$	$Z_{0,1,9}$	$Z_{0,1,10}$
Value	0	0	1	1	1	0	0	0	1	0
Decision Variable	$Z_{0,2,1}$	$Z_{0,2,2}$	$Z_{0,2,3}$	$Z_{0,2,4}$	$Z_{0,2,5}$	$Z_{0,2,6}$	$Z_{0,2,7}$	$Z_{0,2,8}$	$Z_{0,2,9}$	$Z_{0,2,10}$
Value	0	0	0	0	0	0	1	1	0	0
Decision Variable	$Z_{0,3,1}$	$Z_{0,3,2}$	$Z_{0,3,3}$	$Z_{0,3,4}$	$Z_{0,3,5}$	$Z_{0,3,6}$	$Z_{0,3,7}$	$Z_{0,3,8}$	$Z_{0,3,9}$	$Z_{0,3,10}$
Value	1	1	0	0	0	1	0	0	0	1
Decision Variable	$Z_{1,1,1}$	$Z_{1,1,2}$	$Z_{1,1,3}$	$Z_{1,1,4}$	$Z_{1,1,5}$	$Z_{1,1,6}$	$Z_{1,1,7}$	$Z_{1,1,8}$	$Z_{1,1,9}$	$Z_{1,1,10}$
Value	0	0	1	1	1	1	0	1	0	0
Decision Variable	$Z_{1,2,1}$	$Z_{1,2,2}$	$Z_{1,2,3}$	$Z_{1,2,4}$	$Z_{1,2,5}$	$Z_{1,2,6}$	$Z_{1,2,7}$	$Z_{1,2,8}$	$Z_{1,2,9}$	$Z_{1,2,10}$
Value	0	0	0	0	0	0	1	0	1	0
Decision Variable	$Z_{1,3,1}$	$Z_{1,3,2}$	$Z_{1,3,3}$	$Z_{1,3,4}$	$Z_{1,3,5}$	$Z_{1,3,6}$	$Z_{1,3,7}$	$Z_{1,3,8}$	$Z_{1,3,9}$	$Z_{1,3,10}$
Value	1	1	0	0	0	0	0	0	0	1
Decision Variable	$Z_{2,1,1}$	$Z_{2,1,2}$	$Z_{2,1,3}$	$Z_{2,1,4}$	$Z_{2,1,5}$	$Z_{2,1,6}$	$Z_{2,1,7}$	$Z_{2,1,8}$	$Z_{2,1,9}$	$Z_{2,1,10}$
Value	0	0	0	0	1	1	1	0	1	0
Decision Variable	$Z_{2,2,1}$	$Z_{2,2,2}$	$Z_{2,2,3}$	$Z_{2,2,4}$	$Z_{2,2,5}$	$Z_{2,2,6}$	$Z_{2,2,7}$	$Z_{2,2,8}$	$Z_{2,2,9}$	$Z_{2,2,10}$
Value	0	0	1	1	0	0	0	0	0	0
Decision Variable	$Z_{2,3,1}$	$Z_{2,3,2}$	$Z_{2,3,3}$	$Z_{2,3,4}$	$Z_{2,3,5}$	$Z_{2,3,6}$	$Z_{2,3,7}$	$Z_{2,3,8}$	$Z_{2,3,9}$	$Z_{2,3,10}$
Value	1	1	0	0	0	0	0	1	0	1

Similar to model (1) decision variables that are related to RSI values in [70, 100] and [0, 30] intervals are considered.

The decision variables investigation shows these results:

- $Z_{0,1,3}$ is equal to 1. This states that when RSI is in [20, 30] and there isn't an opened trade, opening a buy trade is an optimal decision.
- $Z_{0,2,8}$ is equal to 1. This states that when RSI is in [70, 80] and there isn't an opened trade, opening a sell trade is an optimal decision.
- $Z_{1,1,3}$ is equal to 1. This states that when RSI is in [20, 30] and a buy trade is opened, holding opened buy trade is an optimal decision.
- $Z_{1,2,9}$ is equal to 1. This states that when RSI is in [80, 90] and a buy trade is opened, closing opened buy trade is an optimal decision.
- $Z_{2,1,9}$ is equal to 1. This states that when RSI is in [80, 90] and a sell trade is opened, holding sell trade is an optimal decision.
- $Z_{2,2,3}$ is equal to 1. This states that when RSI is in [80, 90] and a sell trade is opened, closing opened sell trade is an optimal decision.

The mentioned results state that when RSI is above 70, it is probable that exchange rate will decrease soon. So opening a sell trade is optimal decision. Also results show that when RSI is below 30, it is probable that exchange rate will increased soon. So opening a buy trade is optimal decision. These results are equivalent to analysis that is said by Robert D.Edwards, John Magee. (2001). In other words, results of solved model confirm analysis that is said in literature about RSI.

4-3- Results of Model (3)

Model (3) is run to attaining optimal values for SL and TP. After 3 hour running with Lingo 9.0 software, the model didn't reach to a local or global solution. But the upper objective bound of objective function (profit) is attained equal to 37.0115. After that the model is solved for different values for SL and TP. Table 4 shows results for some SL and TP values. Note that because SL shows loss of trades, SL is stated with negative numbers.

Table 4: Values of profits for different values for TP and SL

Strategy Index	1	2	3	4	5	6	7	8	9	10
TP	10	20	30	40	50	60	70	80	90	100
SL	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100
Objective function	33.95663	32.50025	31.1236	28.26472	35.31875	35.16419	30.95311	29.69544	30.23227	30.49944

Note that for all ten proposed strategies Lingo reached to local optimal. But in all strategies upper objective bound is equal to best objective (local objective). So the local optimal is global optimal.

The results of model show that there isn't much different in objective function among different values for SL and TP. Also the maximum value of objective function is 35.31875 and near to upper bound (37.0115).

Also results show that the model (1) with 46.7 for objective function and model (2) with 82.31 for objective function have higher performance relative to model (3). So using of TP and SL has a negative effect on profit.

4-4- Results of Model (4) and Model (5)

Mean of errors of model (4) is high. On the basis of heuristic approach that its description is said in 3.2.2 section, model (5) is developed to solve the lacks of model (4). So in this section results of model (5) are reported.

Then four strong classification algorithms are used for forecasting the direction on the basis of indicators. So in classification algorithms, EMA_{t-i} and RSI_{t-i} ($0 \leq I \leq 15$) are input fields and Target field is D_t. Table 5 shows results of proposed model and four classification algorithms. For use of classification algorithms SPSS Clementine 11.1 software is used.

Table 5: Comparison of Proposed Model (5) with four Classification Algorithms

Methods	Classification algorithms				Proposed model for different values of L				
	C5.0	CART	QUEST	CHAID	L= 80	L= 90	L= 100	L= 110	L= 120
Mean of errors on training data set	40.98%	31.96%	-	38.61%	36.0345%	42.7586%	36.0345%	26.3793%	36.0345%
Mean of errors on testing data set	54.96%	49.62%	-	48.85%	43.511%	42.748%	43.511%	42.748%	45.038%

Table 5 shows that values of objective function (Mean of errors on training data set) for proposed model is equal to lower bound of objective function for L = 80, 100, 110, 120. So in those cases, optimal solution is global solution. Also for L=90, value of objective function is approximately equal to lower bound of objective function (42.0001%). Note that Quest algorithm didn't produce any classification algorithm and is showed with dash line in table 9. Results show that for all values for L, mean of errors of proposed model is less than all of four strong classification algorithms.

Table 6 shows mean of errors that obtained from neural networks. The neural network model is run for different values for nodes in hidden layer. N=1 to N=20.

Table 6: Mean of Errors in Neural Networks Model

N	Mean of Errors on Training data Set	Mean of Errors on Testing data Set
2	43.49	56.49
3	41.75	57.25
4	40.32	48.09
5	43.49	56.49
6	36.67	49.62
7	39.68	51.15
8	42.7	54.2
9	40.32	47.33
10	41.11	57.25
11	41.43	54.96
12	43.49	56.49
13	43.33	54.2
14	42.38	55.73
15	39.84	54.96
16	43.81	56.49
17	42.54	55.73
18	38.41	51.91
19	43.49	56.49
20	42.7	54.2
Average	41.63	54.16

5- Conclusion

This paper had two major sections. First three models were presented. Goal of the models was maximizing profit of trades over a considered period. In those models decision-making was performed on the basis of the RSI value at the start of each day. Results of the models indicated that using Take Profit and Stop Loss parameters don't increase profit of trades. Also results showed that when RSI value is in [70, 100], usually the optimal decision is performing a sell trade. Also the results showed that when RSI value is in [0, 30], the optimal decision usually is issuing a buy trade. In second section a heuristic model for forecasting the direction (increase or decrease) of exchange rate in a day was presented. Then by using of a heuristic approach the accuracy of proposed model was increased. The accuracy of forecasting for proposed model was better than the accuracy of four main classification algorithms.

The novelties of this research are:

- Developing mathematical decision-making models for trading in foreign exchange market
- Results of developed models conform to qualitative contents in literature of Forex about signals of RSI indicator

- Present a heuristic method for forecasting the direction of exchange rate in a day
- Present a heuristic approach for reducing error of forecasting the direction of exchange rate
- Accuracy of proposed models is more than four major classification algorithms

At the end an offer for future research is presented. Forecasting the value of exchange rate changes and forecasting of exchange rate at the end of day can be a subject for another research.

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