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On the Reservation Wages and Liquidity Constraint[1]

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Abstract

This paper argues that a risk averse of workers after-tax reservation wage the difference between her reservation wage and the tax needed to fund the unemployment insurance system when liquidity constraint binds exists and it is unique. The optimality of unemployment insurance based on the responsiveness of reservation wage to unemployment benefit shows the disincentive effect, i.e. higher unemployment benefit will increase workers after tax reservation wage that will make the exit rate lower. This shows that there is a moral hazard problem. The more one tries to protect the worker against unemployment by raising unemployment benefits and funding the benefits by an employment tax, the more selective she becomes.

Keywords: Job Search, Liquidity Constraint, Reservation Wage, Moral Hazard.

1. Introduction

In a recent work, Shimer & Werning (2007) develop a test for the optimality of unemployment insurance based on responsiveness of reservation wages to unemployment benefits. They argue that the after-tax reservation wage measures the well-being of unemployed workers. Clearly any policy that raises the average after-tax reservation wage is beneficial.

They argued that the difference between reservation wage and the tax needed to fund the unemployment insurance system encodes all the relevant information about worker's welfare; they proved this result is true fewer than two financial environments. One that workers has access to financial markets (able to borrow and lend to smooth consumption), only face the budget constraint and the no Ponzi-game

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1. I wish to thank my supervisor Professor Melvyn Coles from whom the author greatly benefited and the seminar participants at University of Essex. All mistakes are of course my responsibility.

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condition and in the second environment, the worker has no access to saving and must consume her income in each period, namely lives hand-to-mouth.

The issue that arises is whether no-Ponzi constraint is a right boundary condition for this class of problem.[1] This paper instead builds a theoretical approach for the optimal level of unemployment insurance using liquidity constraint. I develop a dynamic model of job search with risk aversion where workers confronted by liquidity constraint. Following Shimer & Werning (2007) I assume workers have constant absolute risk aversion preferences. I consider how unemployment workers behave when the liquidity constrained consumption; I find that a worker’s consumption while she/he does not have any asset is equal to unemployment benefit.[2] The worker’s unemployment utility measured in consumption is function of after tax reservation wage given that initial asset is equal to zero.

The literature on responsiveness of unemployment or unemployment duration to unemployment benefits is large, with responsiveness of reservation wages to benefits, Fishe (1982) uses information on actual wages to derive reservation wages and Feldstein and Poterba (1984) use direct survey evidence on reservation wages and discuss the result as evidence of moral hazard cost of raising unemployment benefits. They show that increasing unemployment benefit by 1 may increase pre-tax reservation wages by 0.44.

Following work by Shimer & Werning (2007) on welfare implications of unemployment insurance scheme with worker’s accessibility to financial markets, facing only the budget constraint and the no Ponzi-game condition, I rather introduce liquidity constraint.

The reminder of the paper proceeds as follows: the next section presents my model of sequential search. Section 3 analyses how workers behave when liquidity constraint binds. Section 4 describes the problem of an insurance agency choosing the level of unemployment insurance subject to a budget constraint and solves the problem for the case of binding liquidity constraint. I conclude in

1. The no-Ponzi condition implies that the individual cannot keep borrowing forever. Any debt that has been accumulated eventually has to be paid off.
2. Ruling out savings behaviour is not only unreasonable, Rogerson (1985) shows it also yields distorted policy prescriptions.

section 5.

2. The Model

2.1 The Environment

The model uses a principal/agent framework where a risk neutral Planner insures a risk-averse worker against unemployment risks. I approach this by studying a risk-averse worker in a sequential job search setting (McCall, 1970). Time is continuous and has an infinite horizon, the worker chooses consumption to maximise expected discounted utility:

$$E\int_0^{\infty} e^{-rt}\, u\big(c(t)\big)dt$$

Where $r > 0$ is the worker's rate of time preference, $c(t) \geq 0$ is (flow) consumption at time t and u(.) is an increasing, concave, twice differentiable function with:

$$\lim_{c\to\infty} u'(c) = \infty$$

The worker receives job offers according to a Poisson process with parameter $\alpha > 0$ while unemployed.[1] Corresponding to any job offer is a wage w. Assuming search is random. The offered wage w is considered as a random draw from cumulative distribution function F. Given a job offer the worker both accepts it and becomes employed at wage w, or the worker continues search with no recall. For simplicity assumes the employed worker at wage w remains employed forever; i.e. there is only a single spell of unemployment.[2] While unemployed the worker receives unemployment benefit payment, $b \geq 0$ from the government. Following Shimer & Werning (2007), I assume these benefits paid do not vary with duration and preferences are constant absolute risk aversion (CARA).[3]

While unemployed the worker also chooses consumption optimally. Let A donate the unemployed worker's financial asset. Assuming r also describes the market interest rate then, while

1. Of course in a matching equilibrium, α depends on labour market tightness. Coles (2006) described optimal unemployment policy for the case that workers cannot save but α is determined endogenously.
2. Shavell & Weiss (1979); Hopenhayn & Nicolini (1997); Werning (2002); Kocher-lakota (2004), and Coles (2006a, b) all adopt this approach.
3. To abstract from wealth effects, Shimer & Werning (2007) assumed workers have CARA preferences.

unemployed, A evolves according to the differential equation, where consumption $C^u = C^u(t)$.

$$\frac{dA}{dt} = rA - c^u + b$$

Unlike Shimer & Werning (2007) who assume assets must satisfy the no Ponzi condition[1]. I instead assume unemployed workers are liquidity constrained; i.e. assets must satisfy $A \geq 0$. When re-employed at wage w, the worker is liable to a re-employment tax $\bar{\tau}$.[2] For simplicity assuming this re-employment tax code does not depend on w. Thus when employed, the workers A evolves according to:

$$\frac{dA}{dt} = rA - c^e + w - \bar{\tau}$$

As the worker faces no further risk, the worker's optimal consumption strategy is to consume permanent income from then onwards

$$c^e = rA + w - \bar{\tau}$$

Thus on becoming re-employed with asset A and wage w, the worker enjoys lifetime value:

$$V^e(A, w) = \frac{u(rA + w - \bar{\tau})}{r} \qquad (1)$$

Let $V^u(A, B)$ denote the value of being unemployed with asset A where B= $(b, \bar{\tau})$ denote the Planner's unemployment insurance, UI, Policy. Standard arguments imply the (flow) value of being unemployed is given by the Hamilton-Jacobi-Bellman equation

$$rV^u(A) = \max_c \left[u(c^u) + \frac{\partial V^u}{\partial A}[rA + b - c^u]\right] + \alpha \int_0^{\bar{w}} \max[V^e(A, w) - V^u(A), 0] dF(w) \qquad (2)$$

and subject to the constraint $A \geq 0$. Note that at rate α the worker receives a job offer which either yields capital gain $V^e(A, w) - V^u(A) \geq 0$ (and the worker accepts the job), otherwise the worker

1. They declared that the no-Ponzi condition states that $\lim_{t\to\infty} A(t)e^{-rt} \geq 0$ with probability one (More discussion is in Appendix A).

2. Coles (2006a, b) provided a lump sum tax deduction on re-employment which depends on the length unemployment duration of the completed unemployment spell. Openhayn & Nicolini (1997) implement an income tax premium on future wages

rejects the job offer (and remains unemployed). At each point in time, the worker also chooses consumption $c^u(A)$ to maximise the (flow) value of being unemployed where $u(c)$ is the flow value of consumption and the worker accumulates assets $\dot{A}= [rA + b - c^u(A)]$ which yields corresponding marginal gain in value $\partial V^u(A)/ \partial A$.

3. Optimal Consumption and Reservation Wage Strategy

First I describe the optimal search rule. The worker will accept ant job offer w which satisfies $V^e(A, w) \geq V^u(A)$. As $V^e(A, w)$ is increasing in wages (it is always better to be employed at a firm paying higher wages), the worker's optimal search strategy has the reservation wage, $R(A)$ property: the worker accept wage w if and only if $w \geq R(A)$, where the optimal reservation wage $V^e(R, A) = V^u$ can be written as u(.) is strictly concave.

$$\frac{u(rA+R-\bar{\tau})}{r} = V^u(A) \qquad (3)$$

Then while $A > 0$, the optimal choice of consumption c is given by the first order condition:

$$u'(c^u)=\frac{dV^u(A)}{dA} \qquad (4)$$

Equations (3) and (4) are a pair of policy rules for $c^u(A_t)$ and $R(A_t)$ which describe optimal consumption and optimal reservation wage strategy while unemployed.

$A = 0$ the liquidity constraint binds: To solve this, first consider optimal solution by the worker when unemployed with $A = 0$. The worker consumes $c = b$ and in that case

$$u'(b)=\frac{dV^u(0)}{dA} \qquad (5)$$

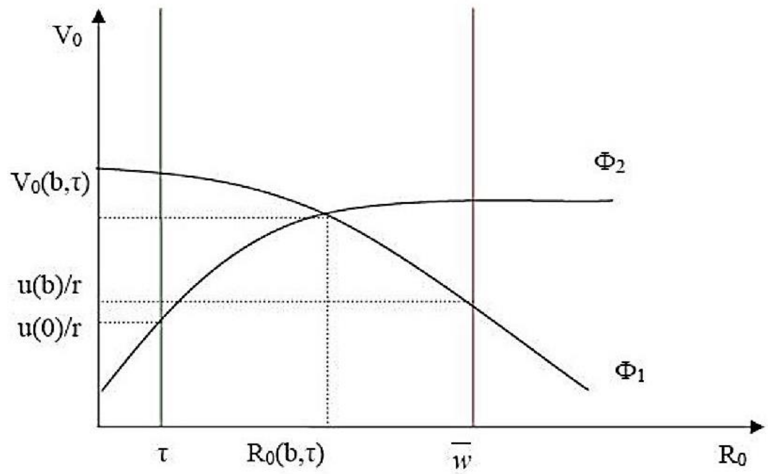
Therefore we have a pair of equations (1), (3) describing V^e, V^u while (5) describes the optimal reservation wage, (3) describes optimal consumption and (6) is a boundary condition for V^u when the liquidity constraint binds. These equations thus determine the worker's optimal search and consumption strategy. I analyse these conditions for the case $A = 0$.

3.1 Consumption and Reservation Wage when (A=0)

Let $R_0 = R(0)$ denote the worker's reservation wage when liquidity constrained and let $V_0 = V^u(0)$. Putting $A = 0$ and c = b in (2), and using (1), (5) implies:

$$Rv_0 = u(b) + \alpha \int_{R_0}^{\overline{w}} \left[\frac{u(w-\overline{\tau})}{r} - V_0\right] dF(w) \qquad (6)$$

Figure 1: Reservation Wage when A=0



R_0 is given by:

$$V^e(0, R_0) = \frac{u(R_0 - \overline{\tau})}{r} = V_0 \qquad (7)$$

(6), (7) are a pair of equations which fully determine V_0at R_0.The following Lemma characterises the solution.

Lemma 1: For any F and b satisfying $0 < b < \overline{w} - \tau$, a pair (V_0, R_0) exists, is unique and $R_0 \in [\tau, \overline{w}]$.

Proof: Integrating equation (6) by parts, noting that $V_0 = u(R_0 - \overline{\tau})/r$, implies:

$$V_0 = u(b) + \alpha \int_{R_0}^{\overline{w}} \frac{u\prime(w-\overline{\tau})}{r} [1 - F(w)]\, d(w) \qquad (8)$$

Thus (8) is an equation for $V_0$where it is a continuous and strictly decreasing function of R_0 for $R_0 < \overline{w}$. Note that at $w = \overline{w}$, (8) is equal to u $(b)/r$. (7) also describes an equation for V_0 and it is a continuous, strictly increasing function of R_0. At $R = \overline{w}$, the condition b$< \bar{w} - \tau$ ensures (8) < (7). While at $R_0 = \tau$ (7) implies (8) is equal to $u(0)/r$ and as (8) implies $(8) \geq u$ (b)/r , we have (7) < (8). Hence by continuity a solution exists, is unique and R_0 satisfies $R_0 \in [\tau, \bar{w}]$, this completes the proof of Lemma 1.

Figure 3.1 depicts the solution to the equations (7), (8). Equation (8) indicates that the value of being unemployed with no asset is increasing in unemployment benefit and employment tax, It is quite clear that increase in b will shift up (8) with no effect on (7) therefore reservation wage R_0will increase. This is the essence of the moral hazard problem in our model –the more one tries to protect the worker against unemployment by raising unemployment benefit and funding the benefit by an employment tax, the more selective she becomes.[1]

Clearly Lemma 1 identifies the solution for V_0 and R_0 when the worker is liquidity constrained.

4. Optimal Unemployment Insurance

The optimal unemployment insurance problem is to choose b, τ and the reservation wage to maximise the worker's value of being unemployed given the worker sets her reservation wage according to the liquidity constraint, $A \geq 0$ and subject to a budget constraint that the cost of unemployment insurance, UI, program is no greater than some exogenous cost C_0 . In other words C_0 . denotes the budget allocated to the insurance program.[2] Let's define:

$$\Psi(A|B) = e^{-\int_0^t (\alpha\left(1-F\left(R\left(A(t);\bar{b},\bar{\tau}\right)\right)\right)ds}$$

Which, conditional on survival, is the probability the unemployed worker remains unemployed at duration t. The budget constraint can then be written as:

$$\int_0^\infty \Psi(A|B)(\bar{b} - \alpha(1 - F\left(R\left(A(t);\bar{b}\,,\bar{\tau}\right)\right)\frac{\bar{\tau}}{r}dt \leq C_0 \qquad (9)$$

where the cost of the program is comprised of benefits outlays minus the opportunity of becoming employed which occurs with arrival rate $\alpha(1 - F(R(A))$ and reduces cost since employed workers pay taxes$\bar{\tau}$.

1. What Mortensen mentioned in his paper discuss the size effect on b and tax when A=0.
2. One can consider the dual problem of minimising the total resource cost which is equal to benefit net of employment tax that delivers a certain level of utility for unemployed worker.

The most natural budget constraint to consider is fair insurance.[1]
For the fair insurance the expected discounted benefit receives while unemployed must be equal to the expected discounted premium while employed. Here, as mentioned before the unemployed worker receives constant b as long as he or she is unemployed and as soon as becoming employed pays lump sum employment tax τ.

4.1 Optimal Unemployment Insurance with A=0

Consider a worker who is unemployed with asset A=0. Since worker's utility is monotone function of R(A) $-\tau$, the problem of planner can be written as:

$$\max_{R_0,\tau,b}(R_0-\tau)$$

subject to:

$$u\,(R_0-\tau)=u(b)+\frac{\alpha}{r}\int_{R_0}^{\bar{w}}[u(w-\tau)-u(R_0-\tau)]\,dF(w)$$ (Incentive Compatibility Constraint)

and:

$$\left[b-\frac{\lambda_0\tau}{r}\right]\int_0^{\infty}e^{-(\lambda_0+r)t}dt=C_0 \qquad \text{(Budget Constraint)} \qquad (10)$$

where $\lambda_0=\alpha[1-F(R_0)]$ Solving (Budget Constraint) for b gives

$$b=(r+\lambda_0)C_0+\frac{\lambda_0\tau}{r} \qquad (11)$$

Substituting (11) into (Incentive Compatibility Constraint) Constraint gives

$$u(R_0-\tau)=u((r+\lambda_0)C_0+\frac{\lambda_0\tau}{r})+\frac{\alpha}{r}\int_{R_0}^{\bar{w}}[u(w-\tau)-u(R_0-\tau)]\,dF(w) \qquad (12)$$

Solving the problem gives:

$$\left[\frac{\lambda_0}{r}-(C_0+\frac{\tau}{r})\frac{\partial\lambda_0}{\partial R_0})\right]=\frac{\alpha}{r}\int_{R_0}^{\bar{w}}\frac{u\prime(w-\tau)}{u\prime(b)}dF(w) \qquad (13)$$

Using the CARA properties and arranging (15) gives[2]:

$$F'(R_0)(rC_0+\tau)=\int_{R_0}^{\bar{w}}(1-\frac{u'(w-\tau)}{u'(b)})\,dF(w) \qquad (14)$$

1. Shimer & Werning (2007) establish the actuarially fair insurance and set $C_0=0$.
2. The proof is in the Appendix B.

$rC_0 + \tau$ shows the disincentive effect, i.e. the higher b will increase R_0 will make λ_0 lower and as a result will make the exit rate lower.

5. Conclusion

This paper characterises optimal unemployment insurance in the McCall (1970) sequential search model when unemployed worker confronted with binding liquidity constraint. While it is important to think about optimal unemployment insurance, the insight is general. For example, Lemma 1, shows that the after tax reservation wage is unique and it is higher than employment tax and less than highest bound of wage. If the liquidity constraint binds her unemployment utility is equal to her after tax reservation wage. The paper has not focused on the optimal timing of unemployment subsides and on the desirability of allowing workers free access to the asset market, going beyond this, the key question is whether the policy towards the unemployed raises the after tax reservation wage? It is obvious increasing unemployment benefit will decrease exit rate from unemployment to employment, there is a moral Hazard problem, i.e. the more tries to protect the worker against unemployment by raising unemployment benefits and funding the benefit by an employment tax, the more selective she becomes.

Appendices

A)

Shimer & Werning (2007) solved the unemployment insurance model by introducing the no-Ponzi condition. Indeed definition of Shimer & Werning (2007) for the no-Ponzi condition is as follows:

"The no-Ponzi condition states that debt must grow slower than the interest rate.

$\lim_{t\to\infty} e^{-rt}a(t) >= 0$, with probability one. Together with the budget constraint $\dot{a} = ra(t) + y(t) - c(t)$, this is equivalent to imposing single present value constraint, with probability one."

Where a(t) represents the asset level and y(t) represents the current income. Let's show the debt of an individual at any single time by D(t) so we can write:

$$D(t) = \int_0^t e^{r(t-t')}\,[c(t') - y(t')]dt'$$

Let's consider the case that worker is unemployed s=u and the unemployment benefit is zero, also c = $\overline{c}$, then:

$$D(t) = \int_0^t e^{r(t-t')}\,[\overline{c}]dt'$$
$$= \frac{\overline{c}}{r}[e^{rt} - 1]$$

So:

$$e^{-rt}D(t) = \frac{\overline{c}}{r} - \frac{\overline{c}}{r}e^{-rt}$$

Of course as $t \rightarrow \infty$ then the $e^{-rt}D(t) \rightarrow 0$. Clearly, one way to rule out the debt to grow unboundedly and not to let a person to die with debt is to prohibit debt entirely, i.e. to require wealth to be always no negative. In a more realistic case that workers are liquidity constrained; i.e. assets A cannot become negative. (An unemployed worker who does not have any asset is unable to borrow against future earnings.)

B)

Clearly by assuming the above objective function $Y(R_0, \tau)$ and the incentive compatibility constraint (12) equal to $g(R_0, \tau)$ and knowing that unemployment insurance should maximize $R_0 - \tau$, then the necessary condition for optimality is:

$$\frac{\frac{\partial Y(.)}{\partial R_0}}{\frac{\partial Y(.)}{\partial \tau}} = \frac{\frac{\partial g(.)}{\partial R_0}}{\frac{\partial g(.)}{\partial \tau}} = -1 \qquad (15)$$

which implies:

$$\frac{\partial g(.)}{\partial R_0} = \frac{-\partial g(.)}{\partial \tau}$$

where $\frac{\partial g(.)}{\partial R_0}$ is equal to:

$$u'(R_0 - \tau) - u'((r + \lambda_0)C_0 + \frac{\lambda_0 \tau}{r})(C_0 + \frac{\tau}{r})\alpha F'(R_0) - \frac{\alpha}{r}\int_{R_0}^{\overline{w}} [u'(R_0 - \tau)]\,dF(w)$$

and also $\frac{\partial g(.)}{\partial \tau}$ is equal to:

$$-u'(R_0-\tau)-u'\left((r+\lambda_0)C_0+\frac{\lambda_0\tau}{r}\right)\frac{\lambda_0}{r}+\frac{\alpha}{r}\int_{R_0}^{\overline{w}}u'(w-\tau)-u'(R_0-\tau)\,dF(w)$$

Here the (15) can be implemented as:

$$u'(b)\left[\frac{\lambda_0}{r}-\left(C_0+\frac{\tau}{r}\right)\alpha F'(R_0)\right]=\frac{\alpha}{r}\int_{R_0}^{\overline{w}}u'(w-\tau)\,dF(w) \qquad (16)$$

We can solve the problem as:

$$[\frac{\lambda_0}{r}-(C_0+\frac{\tau}{r})\frac{\partial\lambda_0}{\partial R_0})]=\frac{\alpha}{r}\int_{R_0}^{\overline{w}}\frac{u'(w-\tau)}{u'(b)}dF(w) \qquad (17)$$

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