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A New Unit Root Test against Asymmetric ESTAR Nonlinearity with Smooth Breaks

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Abstract

This paper proposes a new unit root test against the alternative of symmetric or asymmetric exponential smooth transition autoregressive (AESTAR) nonlinearity that accounts for multiple smooth breaks. We provide small sample properties which indicate the test statistics have good empirical size and power. Also, we compared small sample properties of the test statistics with Christopoulos and Leon-Ledesma (2010) test. The results indicate that our unit root test approach is superior to the test method of Christopoulos and Leon-Ledesma (2010) for both transition parameters (i.e. slow and fast speed), and the test power increases along with the frequency. We apply our test statistics for examining the real interest rate parity hypothesis among OECD countries.

Keywords: Unit Root, Asymmetry, ESTAR, Smooth Breaks, Real Interest Rate Parity.

JEL Classifications: C22, G15.

1. Introduction

This paper develops a new unit root test to allow smooth breaks in the deterministic components and asymmetric nonlinear adjustment. We also extend the Sollis (2009) asymmetric exponential smooth transition autoregressive (AESTAR) nonlinear unit root with smooth breaks by means of a Fourier function. The paper is to set out as follows. Section 2 introduces new unit root test and its construction of

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critical values. Section 3 analyzes the properties of small samples. Section 4 provides an empirical application aiming at the real interest rate parity hypothesis (RIRPH), and final section concludes briefly.

2. Unit Root Test and AESTAR Nonlinearity with Multiple Smooth Breaks

Suppose that a series $\{y_t\}_{t=1}^{T}$ follows the data generating process (DGP) as

$$y_t = \alpha(t) + \varepsilon_t, \tag{1}$$

where

$$\alpha(t) = Z_t\lambda + \sum_{k=1}^{n}\gamma_{1,k}\sin(\frac{2\pi kt}{T}) + \sum_{k=1}^{n}\gamma_{2,k}\cos(\frac{2\pi kt}{T}),$$

$\alpha(t)$ is a time-varying deterministic component. In order to obtain a global approximation from the smooth transition and unknown number, and to equip deterministic components with breaks, we follow Gallant (1981) approach with employing the Fourier approximation and putting both terms of $\sum_{k=1}^{n}\gamma_k\sin(\frac{2\pi kt}{T})$ and $\sum_{k=1}^{n}\gamma_k\cos(\frac{2\pi kt}{T})$ into the model. The reason to select $\sin(\frac{2\pi kt}{T})$ and $\cos(\frac{2\pi kt}{T})$ in the model is based on the fact that a Fourier expression is capable of approximating absolutely integrable functions to any desired degree of accuracy. Where k, T, and t are the number of frequencies of the Fourier function, sample size, and a trend term, respectively, and $\pi = 3.1416$. Z is an optional exogenous regressor which consists of either a constant or a constant with trend term; n denotes the number of frequencies contained in the approximation, and it satisfies $n \le \frac{T}{2}$.

The estimation of equation (1) involves two parameters choice - the choice of n and the choice of k. As noted by Becker et al. (2004), it is reasonable to restrict n=1 since joint null hypothesis of γ s is rejected for one frequency (i.e., $\gamma_{1,k} = \gamma_{2,k} = 0$), and time invariance hypothesis is also rejected. Similarly, Enders and Lee (2012) noted that the restriction n=1 is useful to save the degrees of freedom and prevents over-fitting problem. Hence we re-specify equation (1) as follows:

$$y_t = Z_t\lambda + \gamma_1 \sin(\frac{2\pi kt}{T}) + \gamma_2 \cos(\frac{2\pi kt}{T}) + \varepsilon_t \qquad (2)$$

where $\gamma = [\gamma_1, \gamma_2]'$ measures the amplitude and displacement of the frequency component. Particularly the standard linear specification is a special case of equation (2) while setting $\gamma_1 = \gamma_2 = 0$. There must be at least one of the both frequency components existed if a structural break is appeared. Becker et al. (2004) utilize this property of equation (2) to develop a more powerful test to detect structural breaks under an unknown form than Bai and Perron (2003) test.

In determining an optimal k, we set the maximum of k equal to 5. For any *K=k*, we estimate equation (2) employing ordinary least squares (OLS) method and save the sum of squared residuals (SSR). Frequency *k** is setting as optimum frequency at the minimum of SSR. With above assumption and respect to the deterministic components, we test the following null hypothesis:

$$H_0: \ \varepsilon_t = \upsilon_t, \qquad \upsilon_t = \upsilon_{t-1} + u_t \qquad (3)$$

where u_t is assumed to be an I(0) process with zero mean. To test the null hypothesis, we follow Christopoulos and Leon-Ledesma (2010) to calculate the statistic *via* three steps shown in following.

First step: we set a maximum k equals to 5, and then find out optimal frequency of *k** by employing the methodology described above. We compute the OLS residuals as that:

$$\hat{\varepsilon}_t = y_t - \hat{\alpha}(t) \tag{4}$$

$$\hat{\alpha}(t) = Z_t\hat{\lambda} + \hat{\gamma}_1 \sin(\frac{2\pi k^* t}{T}) + \hat{\gamma}_2 \cos(\frac{2\pi k^* t}{T})$$

Second step: a unit root on the OLS residuals given from equation (4) is tested by using AESTAR model. The AESTAR model combines exponential function and logistic function together, and it assumes the $\hat{\varepsilon}_{t-1}$ as a transition variable to satisfy that

(5)

$$\Delta\hat{\varepsilon}_t = \Omega_t(\eta_1, \hat{\varepsilon}_{t-1})\{S_t(\eta_2, \hat{\varepsilon}_{t-1})\rho_1 + (1 - S_t(\eta_2, \hat{\varepsilon}_{t-1}))\rho_2\}\hat{\varepsilon}_{t-1} + \omega_t \;;\; \omega_t \sim iid(0, \sigma^2)$$

$$\Omega_t(\eta_1, \hat{\varepsilon}_{t-1}) = 1 - \exp(-\eta_1(\hat{\varepsilon}^2_{t-1})) \qquad \eta_1 \geq 0$$

$$S_t(\eta_2, \hat{\varepsilon}_{t-1}) = [1 + \exp(-\eta_2(\hat{\varepsilon}_{t-1}))]^{-1} \qquad \eta_2 \geq 0$$

To test the unit root null hypothesis, unlike the alternative hypothesis of globally stationary symmetric or asymmetric ESTAR nonlinearity with a unit root central regime, we follow Sollis (2009) model to test the null hypothesis H_0: $\eta_1 = 0$ in equation (5) and propose two Taylor approximations, in which one is for exponential function $\Omega_t(\eta_1, \hat{\varepsilon}_{t-1})$ around $\eta_1 = 0$, and the other is for logistic function $S_t(\eta_2, \hat{\varepsilon}_{t-1})$. We propose the following model:

$$\Delta\hat{\varepsilon}_t = \psi_1\hat{\varepsilon}^3_{t-1} + \psi_2\hat{\varepsilon}^4_{t-1} + \sum_{i=1}^{l} \iota_i \Delta\hat{\varepsilon}_{t-i} + \vartheta_t . \tag{6}$$

We include terms $\sum_{i=1}^{l} \iota_i \Delta\hat{\varepsilon}_{t-i}$ to avoid autocorrelation with the error term ϑ_t. The null hypothesis $H_0 : \eta_1 = 0$ in equation (5) becomes to test:

$$H_0 : \psi_1 = \psi_2 = 0. \tag{7}$$

As noted by Sollis (2009), it is not allowed to calculate the standard critical values to test null hypothesis H_0: $\eta_1 = 0$ in equation (5). Hence, this paper tries to compute the finite-sample critical values via

two alternative models. The first model is only assumed an intercept term in equation (2) (i.e., Z= [1]). The second model is equipped an intercept term with trend term (i.e., Z= [1,t]). With setting k between 1 to 5 and sample sizes included 100, 200, 300, and 500 observations, we compute asymptotic critical values by Monte Carlo simulation based on random walk model with pseudo-iid N(0,1) random number and 100,000 replications. The results of critical values are presented in Table 1.

Table 1: Asymptotic Critical Values

Sample size		Model with intercept					Model with intercept and trend				
		k=1	k=2	k=3	k=4	k=5	k=1	k=2	k=3	k=4	k=5
	10%	6.233	5.066	4.654	4.457	4.361	7.991	6.959	6.283	5.969	5.859
T=100	5%	7.348	6.083	5.61	5.383	5.283	9.218	8.145	7.41	7.053	6.91
	1%	9.771	8.352	7.754	7.51	7.363	11.922	10.741	9.984	9.597	9.351
	10%	6.321	5.141	4.723	4.55	4.424	8.102	7.044	6.373	6.069	5.94
T=200	5%	7.415	6.146	5.683	5.469	5.322	9.282	8.207	7.501	7.152	7.001
	1%	9.735	8.436	7.871	7.568	7.396	11.85	10.741	9.941	9.521	9.296
	10%	6.395	5.156	4.774	4.581	4.484	8.121	7.051	6.407	6.107	5.997
T=300	5%	7.472	6.152	5.75	5.5	5.398	9.265	8.178	7.524	7.17	7.008
	1%	9.78	8.396	7.832	7.539	7.394	11.789	10.616	9.934	9.486	9.331
	10%	6.427	5.204	4.775	4.584	4.495	8.161	7.097	6.425	6.149	5.998
T=500	5%	7.48	6.193	5.732	5.496	5.396	9.346	8.24	7.563	7.208	7.036
	1%	9.768	8.324	7.825	7.539	7.403	11.832	10.691	9.933	9.555	9.382

Note: Critical values were calculated using Monte Carlo experiment based on 100,000 replications.

When the null of unit root is rejected against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, we are able to test the null hypothesis of symmetric ESTAR nonlinearity by testing $H_0 : \psi_2 = 0$ against $H_1 : \psi_2 \neq 0$ with a standard *F*-test (*T*-test, or Lagrange Multiplier (*LM*) test).

Third step: when the null hypothesis of unit root is rejected in the second step, we further test whether the nonlinear component in equation (2) is absent. Following Becker et al. (2006), we calculate the F-test statistic:

$$F(k^*) = \frac{(SSR_{restricted} - SSR_{unrestricted}(k^*))/2}{SSR_{unrestricted}(k^*)/T - q}, \quad (8)$$

where $SSR_{unrestricted}$ indicates the SSR with full regression equation (2) and $SSR_{restricted}$ is the SSR without γ s from short regression under the cases of with and without nonlinear component, respectively. Due to the presence of nuisance parameter, the F-test statistic has no standard distribution. We thereby employ the critical values tabulated in Becker et al. (2006).

3. Finite-Sample Size and Power Properties

3.1 Finite-Sample Size

Aiming at the test with a finite-sample size, we consider a following DGP:

$$y_t = Z_t\lambda + \gamma_1 \sin(\frac{2\pi k^* t}{T}) + \gamma_2 \cos(\frac{2\pi k^* t}{T}) + o_t \quad (9)$$
$$o_t = o_{t-1} + \zeta_t \qquad t = 1, 2, ..., T$$

where $\zeta_t \sim iid(0,1)$, $\gamma_1 = \gamma_2 = \{0.1,1\}$, $k^* = \{1,2,3\}$, sample size $T = \{100,300\}$, nominal size is 5%, and 10,000 replications. The values 0.1 and 1 for γ_1 and γ_2 are related to an almost linear and nonlinear process respectively. The results are presented in Table 2, and all reported sizes for these DGPs are close to 5%.

Table 2: Empirical Size of the Test Statistic

Model	Parameter	T=100			T=300		
		k=1	k=2	k=3	k=1	k=2	k=3
Model with intercept	$\gamma_1 = \gamma_2 = 0.1$	0.050	0.051	0.047	0.048	0.047	0.046
	$\gamma_1 = \gamma_2 = 1$	0.053	0.051	0.047	0.050	0.051	0.047
Model with intercept and trend	$\gamma_1 = \gamma_2 = 0.1$	0.054	0.049	0.049	0.048	0.051	0.051
	$\gamma_1 = \gamma_2 = 1$	0.052	0.048	0.051	0.050	0.050	0.046

Note: Nominal size is at the 5% level. Number of replications is 10,000.

3.2. Finite-Sample Power

To investigate the finite-sample power properties from the unit root test, unlike globally stationary process, we consider a following Fourier-AESTAR model as DGP:

$$y_t = Z_t\lambda + \gamma_1 \sin(\frac{2\pi k^* t}{T}) + \gamma_2 \cos(\frac{2\pi k^* t}{T}) + o_t \qquad (10)$$

$$o_t = o_{t-1} + \Omega_t(\eta_1, o_{t-1})\{S_t(\eta_2, o_{t-1})\rho_1 + (1 - S_t(\eta_2, o_{t-1}))\rho_2\}o_{t-1} + \omega_t \; ; \; \omega_t \sim iid(0, \sigma^2)$$

$$\Omega_t(\eta_1, o_{t-1}) = 1 - \exp(-\eta_1(o_{t-1}^2)) \qquad \eta_1 \geq 0$$

$$S_t(\eta_2, o_{t-1}) = [1 + \exp(-\eta_2(o_{t-1}))]^{-1} \qquad \eta_2 \geq 0$$

where all combinations of the parameters and frequencies values are specified to be that $\rho_1 = \{-0.05, -0.1, -0.3\}$, $\rho_2 = \{-0.05, -0.1, -0.3, -0.7, -0.9, -1\}$, $\eta_1 = \{0.1, 1\}$, $\eta_2 = 1$, $\gamma_1 = \gamma_2 = \{0.1, 1\}$, $k^* = \{1, 2, 3\}$, $\omega_t \sim iid(0,1)$, and $T = 300$. The values 0.1 and 1 with respect to η_1 indicate respectively slow and fast transition. The small ρ_2 corresponds to symmetric ESTAR nonlinearity, and the small ρ_1 linking with a big ρ_2 generates the large degree of asymmetric ESTAR nonlinearity.

In addition, we compare our test results with the finite-sample power with the unit root test developed by Christopoulos and Leon-Ledesma (2010). We therefore calculate the critical values with the unit root test of Christopoulos and Leon-Ledesma (2010) for both models, in which one equipped only with intercept, and another contains both intercept and trend terms. The results are presented in Table 3. Clearly, the results indicate (a) our test statistic result is much better than the test model of Christopoulos and Leon-Ledesma (2010) in all cases, especially for the model with intercept and trend terms. (b) When the degree of asymmetry is large, our unit root test is becoming more powerful (almost 20% and 32%, respectively) than the model with both intercept and trend in Christopoulos and Leon-Ledesma (2010). Interestingly, we find that both unit root tests have equal power when symmetric ESTAR nonlinearity exists, and it thereby appears no substantive loss in power if our unit root test is substituted with the test of Christopoulos and Leon-Ledesma (2010).

In addition, the results also show that our unit root test approach is superior to the test method of Christopoulos and Leon-Ledesma (2010) for both transition parameters (i.e. slow and fast speed), and the test power increases along with the frequency.

4. Empirical Application

In this section, we employ our test statistics to re-test the RIRPH among 12 OECD countries. We extend our data set, compared with the examination of Rapach and Wohar (2004), over the period from 1960Q4 to 2012Q3. The nominal interest rate and CPI variables are collected from OECD Economic Indicators. In order to test the RIRPH, we first calculate real interest rate and further use the U.S as benchmark country to compute the real interest rate differentials series according to the following equation (11):

$$(i_t - \pi_t) - (i_t^f - \pi_t^f) = RID_t \qquad (11)$$

where i_t and π_t denote domestic nominal interest rate and inflation rates, respectively, and i_t^f and π_t^f indicate respectively the U.S nominal interest rate and inflation rate. In next step, we test the unit root hypothesis aiming at the RID series with our test statistic when multiple smooth breaks are allowed in intercept model. The empirical results are presented in the Table 4. The significant F statistic in the fourth column indicates that both sine and cosine terms should be included in the estimated model for all countries. Obviously, the joint null hypothesis of unit root test is rejected for all countries except for Canada. Also, the results in column five show that the null of symmetric adjustment for the RID is rejected in Belgium, Netherlands, New Zealand, Norway, and UK.

Table 3: Empirical Power Comparison at the 5% Nominal Level

k^*	ρ_1	ρ_2	Sollis-Fourier model								KSS-Fourier model (Christopoulos and Leon-Ledesma (2010))							
			Constant model				Constant & Trend model				Constant model				Constant & Trend model			
			$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$	
			0.1	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**
1	-0.3	-1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.998	1.000
		-0.7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.999	0.996	0.998
		-0.3	0.996	1.000	0.997	1.000	0.979	0.996	0.980	0.997	0.992	0.997	0.994	0.998	0.964	0.990	0.966	0.992
		-0.1	0.876	0.956	0.879	0.958	0.702	0.874	0.706	0.871	0.818	0.866	0.826	0.870	0.648	0.749	0.655	0.751
		-0.05	0.640	0.800	0.637	0.794	0.436	0.614	0.435	0.617	0.572	0.633	0.560	0.646	0.381	0.482	0.371	0.475
	-0.1	-1	0.995	1.000	0.994	1.000	0.971	0.999	0.971	0.999	0.935	0.932	0.934	0.927	0.877	0.899	0.869	0.894
		-0.7	0.981	0.999	0.982	0.999	0.932	0.994	0.933	0.994	0.915	0.916	0.915	0.913	0.834	0.864	0.823	0.860
		-0.3	0.876	0.958	0.879	0.955	0.703	0.870	0.701	0.869	0.821	0.859	0.828	0.868	0.647	0.757	0.638	0.754
		-0.1	0.520	0.623	0.521	0.626	0.318	0.438	0.323	0.441	0.511	0.609	0.514	0.607	0.318	0.411	0.301	0.409
		-0.05	0.327	0.407	0.331	0.398	0.184	0.241	0.190	0.243	0.321	0.383	0.329	0.402	0.181	0.230	0.186	0.232
	-0.05	-1	0.928	0.992	0.929	0.991	0.817	0.964	0.824	0.965	0.756	0.778	0.762	0.780	0.637	0.716	0.643	0.716
		-0.7	0.866	0.974	0.869	0.973	0.714	0.916	0.713	0.909	0.714	0.747	0.708	0.738	0.559	0.639	0.564	0.639
		-0.3	0.653	0.800	0.642	0.799	0.437	0.617	0.442	0.607	0.566	0.634	0.556	0.631	0.384	0.474	0.382	0.480
		-0.1	0.334	0.408	0.322	0.414	0.197	0.240	0.191	0.245	0.326	0.392	0.326	0.383	0.179	0.229	0.183	0.224
		-0.05	0.207	0.244	0.218	0.252	0.122	0.140	0.118	0.146	0.217	0.253	0.223	0.247	0.116	0.142	0.122	0.136
2	-0.3	-1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000
		-0.7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.998	0.999
		-0.3	0.999	1.000	0.999	1.000	0.993	0.999	0.991	0.999	0.998	1.000	0.998	0.999	0.987	0.996	0.985	0.996
		-0.1	0.958	0.988	0.959	0.986	0.831	0.936	0.825	0.936	0.914	0.921	0.907	0.924	0.767	0.846	0.767	0.842
		-0.05	0.804	0.904	0.800	0.905	0.575	0.753	0.585	0.747	0.711	0.755	0.714	0.752	0.516	0.606	0.516	0.606
	-0.1	-1	0.999	1.000	0.999	1.000	0.988	1.000	0.987	1.000	0.966	0.957	0.963	0.958	0.920	0.925	0.922	0.931
		-0.7	0.997	1.000	0.996	1.000	0.969	0.998	0.968	0.999	0.954	0.948	0.955	0.946	0.896	0.906	0.897	0.907
		-0.3	0.958	0.985	0.961	0.987	0.828	0.936	0.817	0.939	0.912	0.926	0.908	0.924	0.773	0.841	0.766	0.843
		-0.1	0.721	0.801	0.725	0.794	0.465	0.568	0.463	0.580	0.705	0.771	0.707	0.762	0.453	0.556	0.458	0.564

Table 3: Empirical Power Comparison at the 5% Nominal Level

k^*	ρ_1	ρ_2	Sollis-Fourier model								KSS-Fourier model (Christopoulos and Leon-Ledesma (2010))							
			Constant model				Constant & Trend model				Constant model				Constant & Trend model			
			$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$		$\gamma_1=\gamma_2=$	
			0.1	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**	**0.1**	**1**
		-0.05	0.493	0.588	0.504	0.584	0.290	0.361	0.291	0.374	0.489	0.551	0.489	0.533	0.290	0.355	0.294	0.354
	-0.05	-1	0.968	0.996	0.968	0.996	0.903	0.990	0.898	0.991	0.841	0.845	0.841	0.841	0.738	0.796	0.747	0.779
		-0.7	0.937	0.988	0.943	0.988	0.821	0.965	0.828	0.963	0.812	0.820	0.807	0.809	0.678	0.742	0.673	0.741
		-0.3	0.807	0.897	0.803	0.907	0.576	0.754	0.581	0.757	0.710	0.751	0.709	0.755	0.512	0.611	0.516	0.608
		-0.1	0.503	0.585	0.506	0.584	0.286	0.360	0.287	0.368	0.485	0.557	0.483	0.543	0.290	0.355	0.287	0.347
		-0.05	0.345	0.392	0.333	0.394	0.187	0.221	0.190	0.231	0.335	0.383	0.336	0.385	0.186	0.224	0.187	0.222
3		-1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		-0.7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	0.999	0.999
	-0.3	-0.3	1.000	1.000	1.000	1.000	0.997	1.000	0.996	1.000	1.000	1.000	0.999	1.000	0.993	0.998	0.993	0.998
		-0.1	0.975	0.990	0.976	0.992	0.883	0.958	0.886	0.960	0.934	0.945	0.936	0.945	0.844	0.881	0.836	0.883
		-0.05	0.854	0.929	0.847	0.927	0.650	0.809	0.653	0.808	0.763	0.788	0.754	0.783	0.582	0.671	0.595	0.671
		-1	0.999	1.000	0.999	1.000	0.995	1.000	0.994	1.000	0.973	0.966	0.977	0.966	0.941	0.937	0.942	0.946
		-0.7	0.998	1.000	0.998	1.000	0.984	0.999	0.981	0.999	0.967	0.958	0.965	0.962	0.923	0.934	0.926	0.925
	-0.1	-0.3	0.975	0.991	0.973	0.992	0.880	0.959	0.879	0.963	0.940	0.940	0.943	0.946	0.837	0.882	0.835	0.883
		-0.1	0.778	0.837	0.780	0.835	0.541	0.655	0.542	0.658	0.765	0.817	0.763	0.813	0.541	0.646	0.539	0.641
		-0.05	0.544	0.630	0.562	0.632	0.342	0.422	0.342	0.423	0.551	0.605	0.548	0.602	0.340	0.416	0.337	0.414
		-1	0.981	0.998	0.981	0.998	0.926	0.992	0.930	0.992	0.862	0.857	0.862	0.861	0.783	0.812	0.788	0.814
		-0.7	0.960	0.992	0.959	0.992	0.863	0.974	0.871	0.974	0.838	0.837	0.832	0.841	0.730	0.775	0.738	0.778
	-0.05	-0.3	0.852	0.937	0.851	0.933	0.648	0.806	0.655	0.811	0.761	0.783	0.763	0.783	0.595	0.661	0.582	0.669
		-0.1	0.547	0.638	0.552	0.632	0.337	0.424	0.345	0.425	0.550	0.608	0.535	0.603	0.338	0.411	0.341	0.415
		-0.05	0.355	0.415	0.349	0.423	0.212	0.260	0.214	0.263	0.379	0.420	0.364	0.425	0.222	0.267	0.214	0.262

Note: Empirical power were calculated using Monte Carlo experiment based on 10,000 replications and T=300.

Table 4: Empirical Results for RIRPH

Country	Optimum frequency	Optimum lag	F-test statistic	$H_0 : \psi_2 = 0$	$H_0 : \psi_1 = \psi_2 = 0$
Australia	1	4	26.461	-0.72	8.651*
Belgium	5	8	38.651	3.299**	10.968***
Canada	2	5	18.69	1.411	4.445
Denmark	1	4	39.595	1.116	6.488*
France	1	1	20.115	-1.358	12.262***
Ireland	3	4	26.328	0.514	19.773***
Italy	4	4	24.409	0.378	15.66***
Netherlands	5	8	16.267	3.692**	11.021***
New Zealand	1	6	90.656	-2.328**	7.300*
Norway	1	4	28.669	3.279**	10.478***
Switzerland	5	6	19.942	-1.453	6.441***
United Kingdom	4	6	12.36	-2.96**	9.765***

Note: The optimum lag order selected based on the recursive t-statistic. ***, ** and * indicate the null hypothesis is rejected at the 1%, 5% and 10% levels, respectively.

5. Conclusion

In this paper, we generalize the Sollis (2009) AESTAR nonlinear unit root test with allowing multiple smooth temporary breaks by calculating means in Fourier function. The simulation results of Monte Carlo studies also show that our empirical test approach is more reliable and its test statistic is much powerful. Lastly, we further employ our test to examine the real interest rate parity hypothesis over 12 OECD countries and find that the RID series are short-lived, nonlinear mean reversing with multiple smooth breaks, and some adjustments deviated from equilibrium are asymmetrical.

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