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The Stock Returns Volatility based on the GARCH (1,1) Model: The Superiority of the Truncated Standard Normal Distribution in Forecasting Volatility

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Abstract

In this paper, we specify that the GARCH(1,1) model has strong forecasting volatility and its usage under the truncated standard normal distribution (TSND) is more suitable than when it is under the normal and student-t distributions. On the contrary, no comparison was tried between the forecasting performance of volatility of the daily return series using the multi-step ahead forecast under GARCH(1,1) ~ TSND and GARCH(1,1) ~ normal and student-t distributions, until lately, to the best of my understanding. The findings of this study show that the GARCH(1,1) model with the truncated standard normal distribution gives encouraging results in comparison with the GARCH(1,1) with the normal and student-t distributions with respect to out-of-sample forecasting performance. From the empirical results it is apparent that the strong forecasting performances of the models depend upon the choice of an adequate forecasting performance measure. When the one-step ahead forecasts are compared with the multi-step ahead forecasts, the forecasting ability of the former GARCH(1,1) models (using one-step ahead forecast) is superior to the forecasting potential of the latter GARCH(1,1) model (utilizing the multi-step ahead forecast). The results of this study are highly significant in risk management for the short horizons and the volatility forecastability is notably less relevant at the longer horizons.

Keywords: Volatility, Financial Time Series, Truncated Standard Normal Distribution, ARCH/GARCH Models, Forecasting.

JEL Classification: C53, C58.

1. Introduction

In our contemporary world, stock markets represent an essential and active component of the financial markets. Heightened competition in

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the financial markets has increased the significance of prediction of the volatility of stock prices, as evident from several studies conducted over the prior decade. In keeping with the technological advancements, computer programming and data mining techniques extensively employ stock price predictions. In the meantime, it is clear that approaches like artificial neural networks, are utilized as well (Koutrou Manidis et al., 2011). However, dependence on the stock price history and ignorance of other relevant information on market volatility can be understood as the vulnerable points to these approaches.

The statistical analyses on the time series concentrated on the conditional first moment. The expanding role of risk and uncertainty in decision-making models and, in the meanwhile, changes in assessing the risk and volatility measurements over the specified time, enabled the development of new time series methods for the modeling of the second moment, for the analysis of the time series data. The Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models deal with the dependence of the conditional second moment; they also make significant contributions to modeling these processes which are characterized by a high degree of fluctuation. Specifically, they are commonly practiced in the analysis of the financial time series in revealing the heavy-tailed distribution (Teresiene, 2009).

Earlier contributions to the literature which considered the lack of predictive capability of the GARCH models include Tse (1991), Kuen and Hoong (1992), Terasvirta (1996), He and Terasvirta (1999), and Malmsten and Terasvirta (2004). These papers emphasize that the GARCH(1,1) model may not show a better forecasting performance, and does not capture several of the characteristic properties of the financial time series. Goyal (2000) in his investigations on the performance of some GARCH models showed that the GARCH-M (GARCH in the Mean) model exhibits poor out-of-sample forecasting performance when compared with the ARMA specification. Hansen and Lunde (2005) examined 330 different ARCH (GARCH) models to test if any of these models could surpass the performance of the GARCH(1,1) model. They indicated that the GARCH(1,1) model was not superior to any of the more complex models by using exchange data. However, Andersen and Bollerslev (1998); Christodoulakis and Satchell (1998; 2005) highlighted examples of the poor out-of-sample forecasting performance of the GARCH models that is skeptical because of utilizing the squared shocks as a proxy for the true unobserved conditional variance. From studies available in the literature, it is evident that the forecasting ability of the GARCH models has been in question, since the 1990s (see Poon and Granger, 2003).

Hansen and Lunde (2005) revealed that none of the top models possess significantly better forecasting performance than the GARCH(1,1) model. Javed and Mantalos (2013) indicated that the investigation or selection of models for the GARCH models has been explored by many researchers and academicians who concluded that the "performance of the GARCH(1,1) model is satisfactory". Based on these findings, apart from their simplicity and intuitive interpretation, in this study the GARCH(1,1) model was used to predict the volatility and compare the out-of-sample forecasting performances of the different distributional assumptions. The present paper attempted to answer two important questions: (1) Does the GARCH(1,1) model have the ability of forecasting volatility of the squared return series in terms of the out-of-sample performance? (2) Is the use of the GARCH(1,1) model with its truncated standard normal distribution more efficient than the GARCH(1,1) with normal and student-*t* distributions?

Based mostly on the studies of the GARCH(1,1) model, it is assumed that the error term follows the standard normal distribution. However, Mikosch and Starcia (1998) emphasized that the GARCH models with normal standard errors generate a much thinner tail than observed from real data. McFarland et al. (1982) and Baillie and Bollerslev (1991), stated that assuming normality of errors is not reasonable for a variety of applications in financial economics. McNeil and Frey (2000) found that the GARCH models with a heavy-tailed error demonstrate a higher estimating and forecasting performance. Hence, the use of the GARCH models with the student-*t* distribution is considered in a pretty large number of studies (Blattberg and Gonedes, 1974; Bollerslev, 1987; Kaiser, 1996; and Beine et al., 2002). Besides, Vosvrda and Zikes (2004) reported that using the GARCH model with the student-*t* distribution revealed better parameter estimations.

Therefore, the GARCH(1,1) model with its different distributions such as normal, student-*t* and generalized error distribution (GED) were applied in studies by Hsieh (1989), Granger and Ding (1995), Zivot (2008), Koksal (2009) and Vee et al. (2011). While a few of these papers revealed that the GARCH(1,1) with GED exhibited a better forecasting performance than the GARCH(1,1) with the student-*t* distribution, others showed that the GARCH models with the student-*t* distribution fitted better than the GARCH models with the GED distribution. It is evident that the GARCH(1,1)~TSND model can be employed in lieu of the student-*t* distribution. In fact for two reasons it is better to choose the GARCH(1,1)~TSND rather than the GARCH(1,1)~student-*t*. First, it is well recognized that with the student-*t* distribution, determining the degree of freedom of the exponential distribution, or other distributions with a heavy tail is arbitrary. One advantage of the TSND distribution in terms of the student-*t* distribution is that the selection of the degree of freedom is not arbitrary. In the TSND distribution, the shape parameter, a_0 is evident, instead of the degree of freedom. This parameter is selected during the prediction stage of the GARCH(1,1) model by application of the maximum likelihood method. This result indicates that parameter selection in the TSND distribution is not arbitrary like the one in the student-*t* distribution. Secondly, Heracleous (2007) revealed that the GARCH(1,1)~student-*t* provides biased and inconsistent estimations of the parameter, degree of freedom.

Therefore, the this paper aims at showing that the GARCH(1,1) model normally utilized in the literature, has a high level performance for out-of-sample forecasting for the squared returns; the GARCH(1,1)~TSND provides promising results in the out-of-sample forecasting performance when compared with the GARCH ~normal and student-*t*; and it is necessary to employ an accurate forecasting performance measure depending on the characteristics of the return series in order to achieve a good out-of-sample forecasting performance.

The rest of the paper is organized as follows. In Section 2 we introduce the methods and suggest distributional functions. Datasets are described in Section 3. Section 4 reports the empirical findings for both estimation and forecasting, while Section 5 concludes the paper.

2. Methods and Suggested Functional Distribution

This section of the present study investigates in detail the GARCH model and return series used in the prediction of the volatility.

2.1 GARCH Model

The model most frequently used in modeling the financial time series is the GARCH model developed by Bollerslev (1986) instead of the ARCH model, and the particular parameterization also was proposed independently by Taylor (1986). In this model, the conditional variance is the linear function of its own delays, and is represented as given:

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{1}$$

The most common GARCH model in practice is the GARCH(1,1) model. The GARCH(1,1) model indicates the situation in which $p = q = 1$ is clearly shown. The GARCH (p,q) process is weak stationary, if and only if, it satisfies the following condition:

$$\sum\nolimits_{j=1}^{q} \alpha_j + \sum\nolimits_{j=1}^{p} \beta_j < 1 \tag{2}$$

The GARCH process has a constant average and is uncorrelated consecutively. If variance is present, the process is considered weak stationary. The GARCH process can be a strict stationary process without necessarily including a weak stationary characteristic which requires constant average, variance and autocovariance over time. The strict stationarity necessitates the distribution function of any subset of ε_t to remain constant over time. Finite moments are not required for strict stationarity (Yang, 2002).

It is recognized that the GARCH-family models are the ones most widely-used by the researchers who are focused on the financial time series data and forecasting the volatility. In fact, from the existing literature, the GARCH(1,1) model is found to be the most commonly used GARCH process, and constitutes the foundation of several studies in the related literature (Walenkamp, 2008).

According to the studies on forecasting volatility, in terms of the accuracy of the studies of those who desire to work in this field, the forecasting process does exist, which should be followed. The flow chart of the GARCH method is illustrated in Figure1 (Garcia et al., 2005).

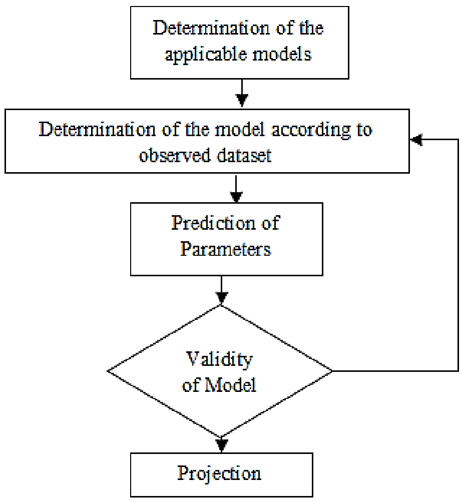

Figure 1: Flowchart of the GARCH Method

2.2 ARCH Model

In this section, the TSND distribution, which exhibits superior heavy-tailed characteristic in comparison to normal distribution, is introduced (Politis, 2004).

The ARCH models introduced by Engle (1982) were designed to capture the volatility-clustering phenomenon in the return series. The ARCH (p) model is described as given below:

$$X_t = Z_t \sqrt{a + \sum_{i=1}^{p} a_i X_{t-i}^2} \tag{3}$$

At this point it is assumed that the $\{Z_t\}$ series is i.i.d. and N(0,1). Nevertheless, the errors $\{\hat{Z}_t\}$ obtained through the ARCH (p) model are not appropriate for the assumption of normality; they exhibit heavy-tailed distribution.

Errors under Equation (3) are obtained as below:

$$\widehat{Z}_t = \frac{X_t}{\sqrt{\widehat{a} + \sum_{i=1}^{p} \widehat{a}_i X_{t-i}^2}} \tag{4}$$

Errors in Equation (4) are essentially expected to behave in the manner observed with the ARCH equation (see Equation (3)) i.i.d. $\widehat{a}, \widehat{a}_1, \widehat{a}_2, ..$ the parameters mentioned above are predictions of the non-negative $a, a_1, a_2, ...$ parameters.

When Equation (3) is considered once more, it can be understood as an operation in which the X_t returns are divided by the standard deviation scale to give them a student-*t* distribution form. Nevertheless, there is no necessity to subtract the X_t's own value from its empirical standard deviation value. Therefore, when the X_t^2 term is included in the transformation process for the student-*t* distribution, the following equation is arrived at:

$$\widehat{W}_t = \frac{X_t}{\sqrt{\widehat{a} + \widehat{a}_0 X_t^2 + \sum_{i=1}^{p} \widehat{a}_i X_{t-i}^2}} \tag{5}$$

Equation (5) is acquired from Equation (6) below (Politis, 2004):

$$X_t = W_t \sqrt{a + a_0 X_t^2 + \sum_{i=1}^{p} a_i X_{t-i}^2} \tag{6}$$

Equation (6) represents the suggested ARCH model. At this point, it is clear that X_t occurs on both sides of the equation. Therefore, Equation (6) can be resolved as indicated below:

$$X_t = U_t \sqrt{a + \sum_{i=1}^{p} a_i X_{t-i}^2} \tag{7}$$

where,

$$U_t = \frac{W_t}{\sqrt{1 - a_0 W_t^2}} \tag{8}$$

It is evident that the ARCH model proposed in Equation (6) is equal to the ARCH (p) model known in Equation (7) and related with a new error term U_t.

If it is assumed that W_t exhibits TSND distribution through variable transformation, the U_t error term in the ARCH model of Equation (7) will have $f\left(u;a_0,1\right)$ density function defined as given below:

$$f\left(u;a_0,1\right)=\frac{\left(1+a_0u^2\right)^{-3/2}\exp\left(-\frac{u^2}{2(1+a_0u^2)}\right)}{\sqrt{2\pi}\left(\Phi\left(1/\sqrt{a_0}\right)-\Phi\left(-1/\sqrt{a_0}\right)\right)},u\in\mathrm{R} \tag{9}$$

where Φ is the standard normal distribution function. Equation (9) is the suggested density function for the ARCH errors (for further details refer to the study by Politis (2004)).

3. Datasets

The dataset used in the present study includes the NASDAQ daily return series extending between 01.03.2000 and 02.27.2013, and the BIST 100 daily return series between 18.01.2006 and 15.03.2013. The data for the NASDAQ daily return series were obtained from http;//finance.yahoo.com, while the data for the BIST 100 daily returns were obtained from the Electronic Data Delivery System (EDDS) of the Central Bank of the Republic of Turkey.

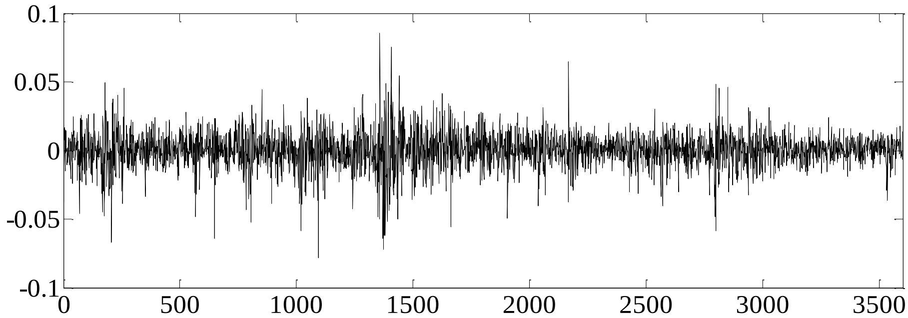

Figure 2: BIST 100 Logarithmic Daily Return Series

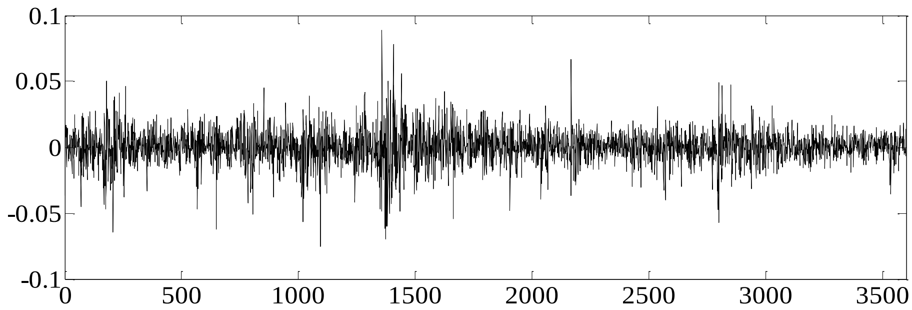

Figure 3: BIST 100 Arithmetic Daily Return Series

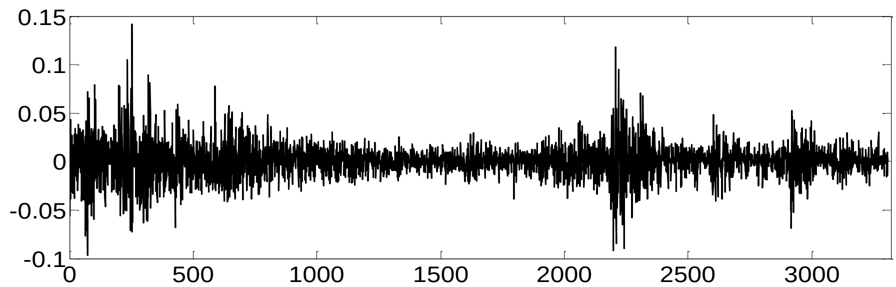

Figure 4: NASDAQ Daily Return Series

The BIST 100 return series are acquired via both logarithmic and arithmetic average methods. In the greater part of the studies available in the relevant literature, it is evident that the return series are calculated using logarithmic formula, whereas the logarithm operation calculates the return rate for the next year smaller than the return rate by the arithmetic formula. This situation can be considered as a different perspective in order to prevent extreme deviations from the observed values. Therefore, in order to study the performance of a suggested model when deviated observations are included in the dataset, the return series were also calculated through arithmetic formula. Descriptive statistics of the return series of three stocks used in the application section of the study are summarized in Table 1, shown below:

From Table 1 it is clear that the kurtosis of all the return series are excessive, whereas, the logarithmic and arithmetic BIST 100 return series exhibited a left-skewed distribution, while the NASDAQ return series exhibited a right-skewed distribution. In the meantime, all the return series were observed to not fit the normal distribution. These results prove that the return series neither possesses the same characteristics nor exhibits normal distribution.

Table 1: Descriptive Statistics of the Return Series

Descriptive statistics/return series	LOGARITHMIC BIST 100	ARITHMETIC BIST 100	NASDAQ
Average	0.000220	0.000300	0.0000843
Median	0.000648	0.000648	0.000650
Maximum	0.085346	0.089094	0.141732
Minimum	-0.078111	-0.075139	-0.096685
Standard deviation	0.012629	0.012607	0.017937
Skewness	-0.358240	-0.246337	0.239071
Kurtosis	7.027062	6.992785	7.944115
Jarque-Bera Test Statistic (likelihood value)	2509.587 (0.000)	2427.759 (0.000)	3400.747 (0.000)
Number of observations	3600	3600	3308

4. Application

As the ARCH models are not expected to successfully forecast the X_t returns, it is anticipated that it is capable of successfully forecasting the X_t^2 squared returns. In spite of all these expectations, some objections have been raised against them in the literature. The opinion normally put forward is that the ARCH/GARCH models exhibit weak out-of-sample-forecasting performances with respect to the daily squared returns (Anderen and Bollerslev, 1998). Further, several works reported that the ARCH and stochastic volatility models revealed weak volatility forecasting capabilities. However, these negative comments are most often connected with forecasting performance measures.

In some instances, it is evident that the condition that the X_t returns have a finite fourth moment is not satisfied.

$V_i^j(Y)$ and $K_i^j(Y)$ represent the empirical variance and kurtosis of the $\{Y_i, Y_{i+1}, ..., Y_j\}$ dataset. While Figure 5 illustrates $V_1^k(X)$ as the k^{th} function of the $X_1, X_2, ..$ data for all the return series, Figures 6 illustrates $K_1^k(X)$ as the k^{th} function of the $X_1, X_2, ..$ data for all the return series.

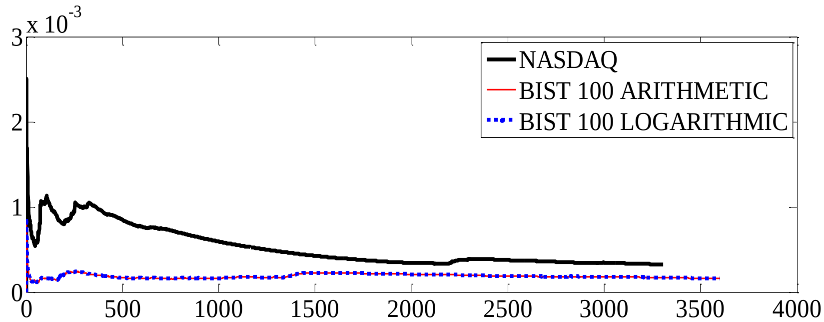

Figure 5: Variance Graphics of the Daily Return Series as the k^{th} Function

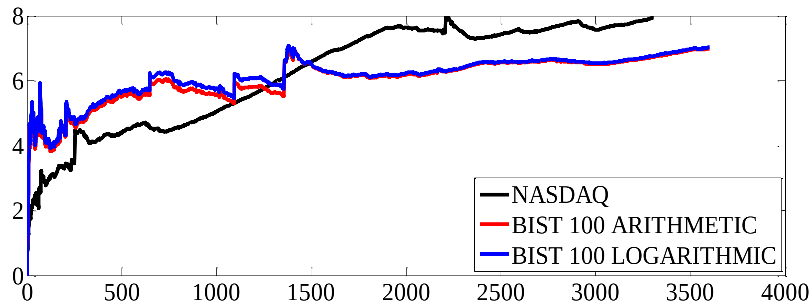

Figure 6: The Fourth Moment Graphics of the Daily Return Series as the k^{th} Function

Figures 5 and 6 show that all the return series would possess a second finite moment, although they may lack the fourth finite moment.

According to the graphics obtained for the three return series, their variances re observed to approach a finite value, but the fourth moment fails to converge to a finite value. Therefore, the Mean Absolute Error and Mean Absolute Scaled Error measures are selected to assess the out-of-sample forecasting performances.

In the next section, first, the parameters of the GARCH(1,1) model are determined under the assumption that the errors have normal distribution, student t distribution and TSND distribution. While the MATLAB software was used for parameter estimations in the GARCH(1,1) model evaluated under the normal and student-*t* distributions, the R-package program was used for parameter predictions under the TSND distribution.

Table 2: Maximum Likelihood Predictions of the GARCH(1,1) Model Obtained through the NASDAQ Return Series

	$\hat{a}_0$	$\hat{A}$	$\hat{B}$	$\hat{C}$
NASDAQ ~ N(0,1)	-	0.059 (0.072)	0.940 (0.924)	6.25e-07 (1.36e-06)
NASDAQ ~ t_v distribution	-	0.057 (0.070)	0.942 (0.928)	6.08e-07 (1.03e-06)
NASDAQ ~ $f(u;a_0,1)$	0.015 (0.029)	0.061 (0.067)	0.937 (0.925)	7.2e-07 (1.08e-06)

Note: The values within parentheses are calculated using 80% of the dataset. We found that the degrees of freedom of the student-t distribution are 39.54056 and 15.99872 for 50% and 80% of the dataset, respectively.

The GARCH(1,1) estimations calculated by utilizing the logarithmic and arithmetic formulae for the BIST 100 return series are presented in Tables 3 and 4, respectively, as shown below:

Table 3: Maximum Likelihood Predictions of the GARCH(1,1) Model Obtained Utilizing the Logarithmic BIST 100 Return Series

	$\hat{a}_0$	$\hat{A}$	$\hat{B}$	$\hat{C}$
BİST 100 ~ N(0,1)	-	0.096 (0.103)	0.871 (0.861)	7.2e-06 (6.5e-06)
BİST 100 ~ t_v distribution	-	0.087 (0.090)	0.885 (0.880)	6.2e-06 (5.22e-06)
BİST 100 ~ $f(u;a_0,1)$	0.063 (0.064)	0.066 (0.067)	0.890 (0.886)	4.6e-06 (3.8e-06)

Note: The values within parentheses are calculated using 80% of the dataset. We found that the degrees of freedom of the student-t distribution are 6.725844 and 6.573482 for 50% and 80% of the dataset, respectively.

The more appropriate approach to assess the results would be by dividing them into two groups, viz., out-of-sample and in-sample groups. The crucial cause for such type of classification is that the performances of the in-sample and out-of-sample predictions are different. As reported by many studies available in the literature, the GARCH(1,1) model performs better with the in-sample group, but performs poorly with the out-of-sample one. Therefore, this section of

Table 4: Maximum Likelihood Predictions of the GARCH(1,1) Model Obtained by Means of the Arithmetic BIST 100 Return Series

	$\widehat{a}_0$	$\widehat{A}$	$\widehat{B}$	$\widehat{C}$
BİST 100 ~ N(0,1)	-	0.095 (0.102)	0.873 (0.863)	7.04e-06 (6.35e-06)
BİST 100 ~ t_v distribution	-	0.087 (0.090)	0.886 (0.882)	6.10e-06 (5.14e-06)
BİST 100 ~ $f(u;a_0,1)$	0.062 (0.063)	0.066 (0.067)	0.890 (0.886)	4.51e-06 (3.77e-06)

Note: The values within parentheses are calculated using 80% of the dataset. We found that the degrees of freedom of the student-t distribution are 6.856274 and 6.687604 for 50% and 80% of the dataset, respectively.

the study focuses on the out-of-sample performance. In the meantime, it is well recognized that the measurements used in comparison with the forecasting performances are effective on the results obtained. For the out-of-sample forecasting performance, the first half of the observation values are used to assess the model parameters, and forecasting is done for the other half of the observed values.

In Table 7, the calculated MAE and MASE values are presented as performance measures. According to the results obtained, it has been noted that the models forecast under the TSND distribution exhibit better out-of-sample forecasting performance when compared with the models forecast under the normal and student-*t* distributions.

To check if the GARCH(1,1) model under TSND shows a higher forecasting performance than the GARCH(1,1) models under normal and student-*t* distributions, the Diebold-Mariano test is used for predictive accuracy (DMt). This facilitates a comparison of the two alternative forecasting models and a predictive likelihood which is also statistical loss function. Table 7 shows the best forecasting performance achieved under the conditional median estimator[1] using the 1-step ahead forecast horizon. Therefore, only the results of the DMt values are noted, as well as the predictive likelihood for the 1-

1. The formula for the conditional median is

$$Median\left(X_{n+1}^2 \mid F_n\right) = \left(a + \sum_{i=1}^{p} a_i X_{n+1-i}^2\right) Median\left(Z_{n+1}^2\right).$$

step ahead forecast obtained using a conditional median estimator (refer Tables 5 and 6).

Therefore, this study has concluded that a statistically significant difference is present among the forecasting performances of the GARCH(1,1) models using the TSND distribution, normal and student-*t* distributions.

Table 5: The Diebold-Mariano Test for Predictive Accuracy

NASDAQ		
DM Test	Nasdaq □ N(0,1)	Nasdaq □ t dist.
Nasdaq □ $f(u;a0,1)$	4.52[3.32e-06] (1.85)[0.03202]	4.59[2.39e-06] (3.96)[4.21e-05]
BIST 100 Logarithmic		
DM Test	Bist 100 □ N(0,1)	Bist 100□ t dist.
Bist 100 □ $f(u;a_0,1)$	5.69[7.49e-09] (4.26)[1.6e-05]	10.264[2.2e-16] (7.24)[5.75e-013]
BIST 100 Arithmetic		
DM Test	Bist 100 □ N(0,1)	Bist 100□ t dist.
Bist 100 □ $f(u;a0,1)$	5.75[5.23e-09] (4.72)[1.40e-06]	10.187[2.2e-16] (7.65)[3.20e-14]

Note: The null hypothesis is the state when both the methods have the same forecast accuracy.

For alternative = "greater", the alternative hypothesis shows that method 2 has greater accuracy than method 1 (GARCH(1,1) □ and the TSND model represent method 2). The values in parentheses are calculated by using 80% of the dataset. The p-values are enclosed within the square brackets.

Table 6: Negative Predictive Likelihood Results

	Conditional Median
	NPL
NASDAQ	
Nasdaq □ N(0,1)	-2.455(-2.508)
Nasdaq □ t dist.	-2.406(-2.448)
Nasdaq □ $f(u;a_0,1)$	**-2.459(-2.526)**
BIST 100 Logarithmic	
Bist 100 □ N(0,1)	-2.528(-2.580)
Bist 100□ t dist.	-2.448(-2.503)
Bist 100 □ $f(u;a_0,1)$	**-2.576(-2.608)**

Table 6: Negative Predictive Likelihood Results

BIST 100 Arithmetic	
Bist 100 □ N(0,1)	-2.528(-2.580)
Bist 100□ t dist.	-2.448(-2.502)
Bist 100 □ f(u;a_0,1)	**-2.578(-2.637**)

Note: The values in parentheses are calculated using 80% of the dataset.
The loss of function is to be minimized. The lower the NPL value,
The higher the forecasting performance.

Table 7: Comparison of the Out-of-Sample Forecasting Performances of the Models

	1-day ahead forecast				***30-days ahead forecast***			
	Conditional Expectation		**Conditional Median**		**Conditional Expectation**		**Conditional Median**	
	MAE	**MASE**	**MAE**	**MASE**	**MAE**	**MASE**	**MAE**	**MASE**
NASDAQ								
Nasdaq □ N(0,1)	1.055	0.780	0.895	0.661	1.076	0.765	0.898	0.664
	(1.092)	(0.809)	(0.903)	(0.670)	(1.158)	(0.859)	(0.916)	(0.679)
Nasdaq □ t dist.	1.056	0.780	0.913	0.675	1.076	0.795	0.920	0.679
	(1.106)	(0.820)	(0.941)	(0.698)	(1.171)	(0.868)	(0.965)	(0.716)
Nasdaq □ f(u;a_0,1)	1.042	0.770	**0.891**	**0.658**	1.055	0.779	**0.894**	**0.660**
	(1.051)	(0.779)	**(0.898)**	**(0.666)**	(1.047)	(0.776)	**(0.899)**	**(0.667)**
BIST 100 Logarithmic								
Bist 100 □ N(0,1)	1.209	0.906	0.912	0.684	1.583	1.186	1.009	0.757
	(1.258)	(0.915)	(0.915)	(0.670)	(1.782)	(1.305)	(1.046)	(0.766)
Bist 100□ t dist.	1.205	0.903	0.992	0.743	1.554	1.165	1.175	0.880
	(1.236)	(0.905)	(1.004)	(0.736)	(1.674)	(1.225)	(1.227)	(0.899)
Bist 100 □ f(u;a_0,1)	1.062	0.780	**0.894**	**0.670**	1.094	0.820	**0.906**	**0.679**
	(1.069)	(0.783)	**(0.888)**	**(0.650)**	(1.109)	(0.812)	**(0.895)**	**(0.655)**
BIST 100 Arithmetic								
Bist 100 □ N(0,1)	1.209	0.906	0.913	0.684	1.584	1.187	1.100	0.757
	(1.257)	(0.921)	(0.915)	(0.670)	(1.779)	(1.303)	(1.045)	(0.766)
Bist 100□ t dist.	1.207	0.905	0.992	0.743	1.564	1.172	1.177	0.882
	(1.244)	(0.911)	(1.007)	(0.737)	(1.714)	(1.255)	(1.245)	(0.912)
Bist 100 □ f(u;a_0,1)	1.058	0.793	**0.894**	**0.670**	1.086	0.814	**0.904**	**0.678**
	(1.067)	(0.781)	**(0.888)**	**(0.650)**	(1.105)	(0.810)	**(0.895)**	**(0.655)**
	60-days ahead forecast				***90-days ahead forecast***			
	Conditional Expectation		**Conditional Median**		**Conditional Expectation**		**Conditional Median**	
	MAE	**MASE**	**MAE**	**MASE**	**MAE**	**MASE**	**MAE**	**MASE**
NASDAQ								
Nasdaq □ N(0,1)	1.098	0.811	0.902	0.667	1.121	0.828	0.908	0.671
	(1.227)	(0.909)	(0.932)	(0.691)	(1.292)	(0.826)	(0.951)	(0.691)
Nasdaq □ t dist.	1.097	0.810	0.927	0.685	1.118	0.826	0.936	0.691
	(1.241)	(0.920)	(0.994)	(0.737)	(1.311)	(0.972)	(1.024)	(0.758)

Table 7: Comparison of the Out-of-Sample Forecasting Performances of the Models

Nasdaq □ f(u;a$_0$,1)	1.070 (1.048)	0.790 (0.777)	**0.898** **(0.903)**	**0.663** **(0.669)**	1.084 (1.051)	0.801 (0.779)	**0.903** **(0.908)**	**0.667** **(0.673)**
BIST 100 **Logarithmic**								
Bist 100 □ N(0,1)	1.743 (1.979)	1.307 (1.449)	1.059 (1.105)	0.794 (0.809)	1.804 (2.046)	1.352 (1.498)	1.079 (1.125)	0.809 (0.824)
Bist 100□ t dist.	1.728 (1.870)	1.295 (1.370)	1.272 (1.338)	0.953 (0.980)	1.805 (1.952)	1.353 (1.430)	1.316 (1.385)	0.986 (1.014)
Bist 100 □ f(u;a$_0$,1)	1.110 (1.122)	0.832 (0.822)	**0.913** **(0.899)**	**0.684** **(0.658)**	1.114 (1.125)	0.835 (0.824)	**0.914** **(0.900)**	**0.685** **(0.659)**
BIST 100 **Arithmetic**								
Bist 100 □ N(0,1)	1.750 (1.981)	1.312 (1.451)	1.061 (1.105)	0.795 (0.809)	1.815 (2.052)	1.360 (1.503)	1.081 (1.127)	0.811 (0.825)
Bist 100□ t dist.	1.747 (1.939)	1.310 (1.420)	1.279 (1.372)	0.958 (1.004)	1.832 (2.038)	1.373 (1.493)	1.326 (1.429)	0.994 (1.046)
Bist 100 □ f(u;a$_0$,1)	1.100 (1.118)	0.825 (0.819)	**0.911** **(0.899)**	**0.683** **(0.658)**	1.104 (1.121)	0.828 (0.821)	**0.913** **(0.900)**	**0.684** **(0.659)**

Note: The values in parentheses are calculated using 80% of the dataset. The sample variance used is the Benchmark. The formula of the m-step ahead forecast of the conditional variance of the GARCH(1,1) models is given:
$E\left(\sigma_{n+m}^2 \mid I_n\right) = \alpha_0 + \left(\alpha_1 + \beta_1\right) E\left(\sigma_{n+m-1}^2 \mid I_n\right)$. I_n is the information set available at time n.

5. Conclusions

This paper has attempted to re-examine the forecasting ability of the ARCH/GARCH models in the context of distributional function meaning TSND, and the forecasting performance measures like MAE and MASE.

It is a known fact that in the volatility prediction for the return series, the error distribution types that are largely preferred are the ones exhibiting heavy-tailed characteristics such as normal, student-*t* and GED distributions. It is clear that the distributions of the error predictions obtained after forecasting the volatility of the stock return series using the ARCH and GARCH models, relying on the assumption of errors are normally distributed, and do not conform to the normal distribution in reality. This finding reveals that the error distributions exhibit more heavy-tailed characteristics compared with the normal distributions; thus, it becomes essential that the errors be considered under different distributions which display heavy-tailed characteristics while modeling the volatility of the return series.

While selecting the distributions having heavy-tailed characteristics, the basic difficulty lies in determining the degree of freedom. Therefore, the selection of the degree of freedom for distributions with heavy-tailed characteristics arbitrarily necessitates considering a model that exhibits more heavy-tailed characteristics compared with the normal distribution in volatility, modeling of the stock return series and evaluation of out-of-sample forecasting performances. Further, this distribution, which does not demand arbitrary degree-of-freedom, and which demonstrates better performance compared with the normal distribution, and which has at least the same or better performance compared with the student-*t* distribution, is termed as TSND distribution. The distribution shape parameter is denoted by a_0. This parameter is predicted using the pseudo-likelihood method. This result reveals that parameter selection is not arbitrary.

In order to prove that the forecasting performance of the GARCH(1,1) model is better under the TSND distribution compared with the forecasts under normal and student-*t* distributions, both the NASDAQ and BIST 100 return series calculated by logarithmic and arithmetic formulas were used. All the three return series reveal different characteristics and include different observation numbers. On the contrary, the studies which report the weak forecasting performance of the GARCH(1,1) model recognize that when good forecasting performance measures like MAE and MASE are used, an acceptable out-of-sample forecasting performance is exhibited. In the case where the return series lacks a finite fourth moment, the selection of the MAE or MASE measure, frequently used in the literature, is correct in terms of the squared returns.

While it has been observed that the forecasting accuracy of GARCH(1,1)~TSND model, suggested in terms of out-of-sample performance, is superior to the forecasting accuracy of GARCH(1,1) model using normal and student-*t* distributions. Another significant fact is that the absence of any difference between the forecasting volatility of the return series calculated by logarithmic or arithmetic formula. As the coefficient estimations obtained are similar or very close to each other, no difference in terms of out-of-sample forecasting performances is observed.

The TSND distribution shape parameter displays similarities to the t_v-distribution with v degree of freedom, according to the different values of a_0. For instance, when $a_0 = 0.1$, the TSND distribution reveals a distribution very close to the *t*-distribution with 5 degrees of freedom. However, it is noted that the distribution tail shows slightly thinner tail characteristics with respect to the t_5-distribution. At the interpretation stage of the results obtained, in order to avoid any biased assessment, the degree of freedom of the *t*-distribution is determined by the MATLAB software, used in the estimation of the model coefficient.

Several studies in the literature have reported that the resulting forecasting performances will be effective if the dataset is distinguished into two sections when out-of-sample forecasting performances are assessed; while the first section is used to estimate the parameters of the models, the second section is considered to determine the forecasting performance. Only half of the dataset is used for forecasting. However, because the characteristics of the datasets considered do not bear any of the characteristics of the real dataset, in the second stage 80% of the dataset is considered while the remaining 20% is used for forecasting. The findings obtained imply that by increasing number of observations in the dataset, otherwise referred to as the training set, and using them in the estimation parameters of the models, induces a rise in the MAE and MASE values used in the estimation of the forecasting performances, thus facilitating the acquisition of the results that concur with the results in the literature. In the meantime, the occurrence of a rise in the MAE values was determined for 20% of the dataset forecasted, based on the coefficients estimated, by considering 80% of the dataset for both the GARCH(1,1)~ND and the GARCH(1,1)~t_v models. These results reveal that the GARCH models exhibit an excellent out-of-sample forecasting performance when the fitting forecasting performance measure is used.

The main thrust is that researchers or practitioners must exhibit great care when determining the sample size for the training set. They need to select a reasonable forecasting performance measure and utilize one model under several distributions to forecast the volatility present in different datasets.

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