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Modeling Gold Volatility: Realized GARCH Approach

Esmaiel Abounoori*[1], Mohammad Amin Zabol[2]

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Abstract

Forecasting the volatility of a financial asset has wide implications in finance. Conditional variance extracted from the GARCH framework could be a suitable proxy of financial asset volatility. Option pricing, portfolio optimization, and risk management are examples of implications of conditional variance forecasting. One of the most recent methods of volatility forecasting is Realized GARCH (RGARCH) that considers a simultaneous model for both realized volatility and conditional variance at the same time. In this article, we estimate conditional variance with GARCH, EGARCH, GIR-GARCH, and RGARCH with two realized volatility estimators using gold intraday data. We compared models, for in-sample fitting; by the log-likelihood value and used MSE and QLIKE lose functions to evaluate predicting accuracy. The results show that the RGARCH method for GOLD outperforms the other methods in both ways. So, using the RGARCH model in practical situations, like pricing and risk management would tend to better results.

Keywords: Realized GARCH, Gold, GARCH Models, Volatility.
JEL Classification: G10, G15, G17.

1. Introduction

Examples of volatility prediction applications include option pricing, optimal portfolio selection, and risk management. Variance, standard deviation, conditional variance, or all of these indicators, is, in fact, a proxy for volatility and a tool for its estimation. Today, in many types of research, the conditional variance estimated from GARCH family models has used a reasonable estimate of volatility.

In conventional GARCH models, only daily stock returns are used to predict daily conditional variance. Since the information obtained from daily returns is lower than the different criteria derived from

1. Department of Economics, Semnan University, Semnan, Iran (Corresponding Author: esmaiel.abounoori@semnan.ac.ir).
2. Department of Economics, Semnan University, Semnan, Iran (m.zabol@semnan.ac.ir).

intraday data, the information set of conventional GARCH models is limited. In addition, since GARCH models are based on moving average with decreasing weight, these models act a bit slow to react to volatility changes (Andersen et al., 2003). Thus, there was a tendency to introduce the intraday criteria in the framework of GARCH models. Engel (2002) proposed to include realized variance as an exogenous variable in GARCH models. The disadvantage of this innovation is that only one-day ahead conditional variance prediction is possible. Recently, Hansen et al. (2012) presented a model called “realized GARCH”, which provides a framework for modeling the realized volatility and conditional variance at the same time in one model.

In this paper, using the Gold five-minutes intra-day data from April 2012 to April 2018, we compare the realized GARCH model with some conventional GARCH models such as GARCH, EGARCH, and GJR-GARCH. The comparison will be in two ways. First, we consider how well data has been fitted in the models. Then, we examine the accuracy of the prediction of the conditional variance of the sample by using the rolling window approach and using a loss function to select the most accurate model.

2. Literature Review

The GARCH models are widely used in finance. Concerning the case of option valuation, the latest studies have been done by Badzko et al. (2015) and Huang et al. (2017); the latest study showed that using the GARCH model for the S&P index is more appropriate than another volatility method in the case of option pricing. Regarding portfolio optimization, GARCH is used by Ðrnovkiewicz et al. (2016) and Sahamköping et al. (2018).

In order to explain how the GARCH models emerge, we should put aside the heteroscedasticity assumption of the linear regression. Engel (1982) introduces a particular type of heteroscedasticity in which the variance of the innovation term is a function of the lags of squared innovations. The introduction of this kind of heterogeneity variance provided a very important tool for economists and especially researchers in the field of financial econometrics to measure and estimate the conditional variance of a series. Consider an AR(1) process for asset returns as in equation (1).

$$r_t = \mu_0 + \mu_1 r_{t-1} + \varepsilon_t \tag{1}$$

Where r_t represents the asset returns and ε_t is an iid with zero means. In this case, the conditional variance of ε_t may vary over time and be a function of the previous innovations. This model was originally presented by Engel (1982). The reason for introducing this model was that although it is observed, ε_t are independent, but their square is related to each other. Engel suggested the following equation for the conditional variance ε_t, which is known as ARCH (p).

$$\sigma_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + \ldots + a_p\varepsilon_{t-p}^2 \tag{2}$$

The number of the lag of an ARCH model, p, is large, leading to an increase in the number of estimated parameters. As a result, Bollerslev (1986) suggested the following model overcome this problem.

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i\varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j\sigma_{t-j}^2 \tag{3}$$

Where a_i and b_j are assumed to be positive to ensure that the variance is positive. This model is known as the Generalized ARCH model, GARCH (p, q). If q is zero, then this model will be reduced to ARCH(p).

Since in the GARCH model, ε_t appear in squared form in the equation, the sign of these shocks does not affect the conditional variance. Meanwhile, it has been observed that negative shocks or bad news increase the variance more than the positive shocks or good news. Also, the parameters of a GARCH model should be restricted to ensure that the variance stays positive. For this purpose, Nelson (1991) introduced the EGARH model as follows.

$$\begin{aligned} h_t &= a_0 + \sum_{i=1}^{p} a_i \frac{|\varepsilon_{t-i}| + \gamma_i\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} b_j h_{t-j} \\ h_t &= \ln\sigma_t^2 \end{aligned} \tag{4}$$

In this model, when ε_t is positive, the total shock effect is$(1+\gamma_t)\varepsilon_t$, and if there is bad news, the total shock effect will be as large as$(1-\gamma_t)\varepsilon_t$. If bad news is supposed to have a higher variance, we expect γ to be negative. Apart from the fact that the effects of good and bad news are considered differently invariance, this model has the advantage of the GARCH model, which variance will always be positive for any coefficients.

Another way to consider the effect of good and bad news on the variance is by using a dummy variable as follows:

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \gamma_i S_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2 \quad (5)$$

In which S_{t-i} is a dummy variable that is zero if ε_{t-1} is positive, and 1 if ε_{t-1} is negative. In this case, the effect of a positive shock is $a_i\varepsilon_{t-i}^2$and the effect of a negative shock is$(a_i+\gamma_i)\varepsilon_{t-i}^2$. By assumption of the effect of bad news on the variance, we expect γ_i to be positive. This model is known as GJR introduced by Glosten, Jagannathan, and Runkle (1993).

After these initial models, a number of models and explanations were introduced for conditional variance modeling. Some researchers tried to use another explanatory variable apart from the squared innovation in the model, which became known as GARCH-X models. In 2003, Engle first used the realized variance criteria to explain the conditional variance. This effort, although was an improvement of the GARCH models, but actually, it was the same GARCH-X specification. In fact, the realized variance was added as an exogenous variable to the GARCH model.

An important problem of using realized volatility in a GARCH-X framework is that the models only can predict variance for the next period. Engel and Gallo (2006) introduced the first model in which a structure was presented for realized volatility so that an additional latent volatility process was considered for each realized measure in the model. This model is known as the Multiplicative Error Model.

Another model concerning realized volatility as an endogenous variable is the HEAVY model presented in Shepard and Sheffard

(2010), which as a mathematical view is a nested MEM model. Unlike traditional GARCH models, these models operate on the basis of multivariate latent volatility processes. For example, three latent volatility processes are used in a MEM model, but the HEAVY model includes at least two latent volatility processes.

In one of the most recent works done in this regard, Hansen et al. (2012), introduced the third equation into a GARCH model to simulate the realized volatility in an Endogenous way. They consider conditional variance as a function of realized volatility as follows:

$$r_t = \sqrt{h_t}\varepsilon_t \tag{6}$$

$$h_t = \omega + \beta h_{t-1} + \gamma x_{t-1} \tag{7}$$

$$x_t = \xi + \varphi h_t + \tau(\varepsilon_t) + u_t \tag{8}$$

In which x_t is the realized volatility, h_t denotes the conditional variance, and τ (.) is the leverage function. Equation (6) and (7) represent the mean and conditional variance, respectively as in conventional GARCH models replacing equation (8) in equation (7). Adding regression (8) distinguishes realized GARCH models from conventional GARCH models. This equation is known as the measurement equation because it relates the realized measure to the hidden volatility (Hansen et al., 2012). Leverage function in the simplest form could be zero but Hansen (2012) suggest$\tau(\varepsilon_t) = \lambda_1(\varepsilon_t) + \lambda_2(\varepsilon_t^2 - 1)$, so this function can capture asymmetric behavior of shocks too. This model is known as the RGARCH model. Tian et al (2015) used this method and traditional GARCH methods to estimate the daily volatility of the short-term interest rate in the euro–yen market. The results have indicated that the RGARCH model has better performance than traditional GARCH models regarding the prediction of the conditional variance. Sharma & Vipul (2016) investigate the variance predict the performance of the RGARCH model for 16 stock indices in a 14-year period. They indicated that the results are sensitive to the performance decision criteria. Wei et al. (2017) construct a new realized GARCH model by introducing the perturbation of leveraged parameter in the volatility equations of the realized GARCH model. The empirical analysis of the high-frequency data of the Shanghai Stock Exchange 50 index shows that using their new model can improve the prediction accuracy of a measure of risk to a certain extent. Huang et al. (2017) conduct an extensive empirical analysis on S&P500 index options using realized GARCH and the results show that their computationally fast formula outperforms competing methods in terms of pricing errors, both in-sample and out-of-sample. Jiang et al. (2018) apply realized GARCH models by introducing several risk measures of intraday returns into the measurement equation, to model the daily volatility of E-mini S&P 500 index futures returns. The empirical results show that realized GARCH models using the generalized realized risk measures provide better volatility estimation for the in-sample and substantial improvement in volatility forecasting for the out-of-sample.

Although we expect that the RGARCH model outperforms other GARCH models, but there is some evidence that it depends on the type of asset. In this paper, we intend to examine the performance of the RGARCH model in predicting the conditional variance of GOLD in comparison with other GARCH family models.

3. Methodology

In order to estimate a realized GARCH model, intraday data should be used so the realized volatility which is needed in this procedure could be calculated. So, we have used five-minutes gold trades in forex (XAU/USD)[1]. The data has been gathered from April 2012 to April 2018.

In GARCH models, there is a mean equation, which in this paper is considered as an AR(1) prose.

$$r_t = \mu_0 + \mu_1 r_{t-1} + \varepsilon_t \tag{9}$$

In which r_t is the daily return on financial assets. This specification of the model makes it possible to assume that the return is related to a constant amount, along with a coefficient of return on

1. The data is available in many forex source such as link below: https://www.dukascopy.com/swiss/english/marketwatch/historical/

the trading day before and the shock of that day. We also consider equations (10) to (12) for the estimation of the traditional GARCH models as the following;

GARCH(1,1) $$\sigma_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + b_1\sigma_{t-1}^2 \quad (10)$$

EGARCH(1,1) $$h_t = a_0 + a_1\frac{|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1}}{\sigma_{t-1}} + b_1 h_{t-1} \quad , h_t = \ln\sigma_t^2 \quad (11)$$

GJR-GARCH(1,1) $$\sigma_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + b_1\sigma_{t-1}^2 \quad (12)$$

And RGARCH as follows;

$$\begin{aligned} h_t &= \omega + \beta h_{t-1} + \eta x_{t-1} \quad , h_t = \ln\sigma_t^2 \\ x_t &= \xi + \varphi h_t + \tau(\varepsilon_t) + u_t \\ \tau(\varepsilon_t) &= \lambda_1\varepsilon_t + \lambda_2(\varepsilon_t^2 - 1) \end{aligned} \quad (13)$$

In which x_t is the realized volatility, h_t denotes the conditional variance, and τ (.) is the leverage function that relates the volatility to variance equation which distinguishes realized GARCH models from conventional GARCH models. This equation is also known as the measurement equation because it relates the realized measure to the hidden volatility (Hansen et al.. 2012). We used two proxies of realized volatility which are RV and BV. So we mention RV-GARCH and BV-GARCH as the realized GARCH models with using RV and BV proxies of the realized volatility respectively. These two proxies can be calculated as follows:

$$RV_t = \sum_{i=1}^{M} r_{i,t}^2 \quad (14)$$

$$BV_t(\Delta) = \mu^{-2}\frac{M}{(M-1)}\sum_{i=2}^{M}|r_{t,i}||r_{t,i-1}| \quad (15)$$

Where $r_{i,t}$ represents the ith intraday return in day t, M denotes the number of trades in a day and μ is equal to $\sqrt{2/\pi} \cong 0.79788$. One important difference between RV and BV is that the first is not robust to jumps but the latter is.

A good model not only should fit data well but also it should have more accurate performance in predicting out of sample volatility. So in this paper both in-sample and out of sample modeling performance has been investigated. For in-sample fitting, as the method of estimation is ML, the Log L value is a reasonable measure to compare models.

To compare the predictive performance of conventional GARCH models to the realized GARCH model, the rolling window technique of size 500 has been used. Therefore we used the first 500 data to predict the conditional variance of day 501 and compared it with the actual volatility of the same day. Then we omit the first day and add day 501 to predict volatility and compare it with actual one of day 502 and so on.

we used the method of Hansen et al. (2005) to estimate the actual volatility as follow:

$$\sigma_t^2 = \hat{c}.RV_t \quad , \hat{c} = \frac{n^{-1}\sum_{t=1}^{n}(r_t - \hat{\mu})^2}{n^{-1}\sum_{t=1}^{n}RV_t} \qquad (16)$$

Where n is the number of the trading day, x_t is the daily return and $\hat{\mu}$ denotes the average of daily returns in n days. After defining the actual volatility, a loose function should be used in order to rank the models by the accuracy of predicted volatility. Patto (2011) showed that among 9 loose functions to rank the volatility, only MSE and Qlike are robust in the possible existence of proxy error and these two only should be used. These equations are as follow:

$$\mathrm{MSE} = \mathrm{E}\left(l_{1,k,t}\right) \quad , \quad l_{1,k,t} = (\sigma_t^2 - \hat{\sigma}_t^2)^2 \qquad (17)$$

$$\mathrm{QLIKE} = \mathrm{E}\left(l_{2,k,t}\right) \quad , \quad l_{2,k,t} = \log\left(\hat{\sigma}_t^2\right) + \frac{\sigma_t^2}{\hat{\sigma}_t^2} \qquad (18)$$

The lower lose function, the better model and among all models, the model with the lowest value in the loose function is the most accurate model.

4. Empirical Results and Discussion

In order to estimate the model, we have used five-minutes gold trades in forex (XAU/USD) from April 2012 to April 2018. Table 1 shows the daily GOLD return in which indicates that 90% of the returns are between -1.3% and 1.3%. Kurtosis and Skewness calculated from the winsorized gold return at 1^{st} and 99^{th} percentiles[1] suggest that the center part of the returns distribution doesn't differ much from the normal distribution.

Table 1: Descriptive Statistics of Daily GOLD Return in Full Sample

n	smallest	5^{th} percentile	mean	median	95^{th} percentile	largest	std. dev.	Skewness*	Kurtosis*
1941	-9.50%	1.30%	0	0	1.30%	4.90%	0.86%	-0.04	3.97

Note: Kurtosis and Skewness are calculated from the winsorized gold return at 1^{st} and 99^{th} percentiles**.**

The predictive performance of GARCH models should be investigated in both in the sample and out of sample data. Table 2 represents the log-likelihood of GARCH models. As the logL for RV-GARCH is bigger than other models, it means the RV-GARCH model fits the gold data better.

Table 2: LogL Value of the Models Estimated in Full Sample

MODEL	Log L	Rank
GARCH	-1685.792	4
EGARCH	-1685.858	5
GJR-GARCH	-1685.789	3
RV-GARCH	-1676.404	1
BV-GARCH	-1677.098	2

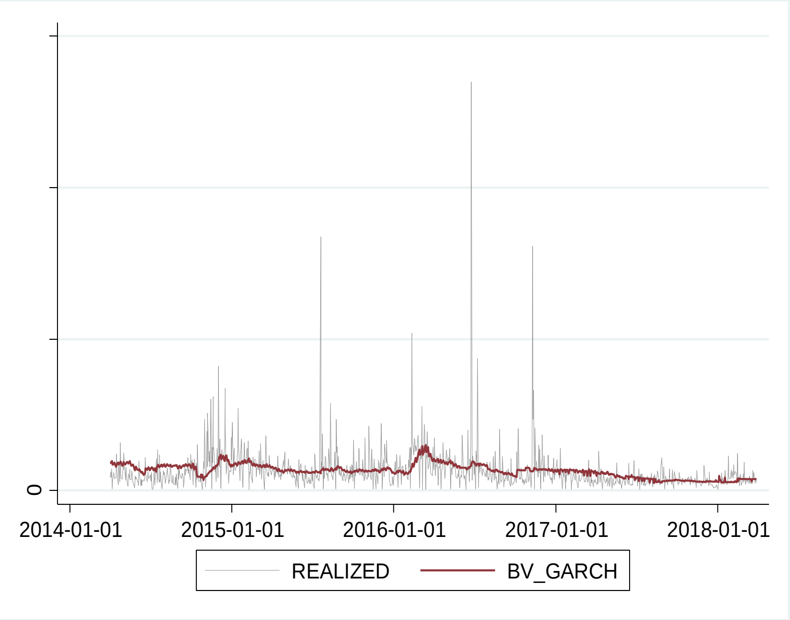
For out of sample performance evaluation of GARCH models, one-step-ahead conditional variances of the models were predicted and the value of loose functions for each model was calculated. The

1. Winsorised the data means data less than 1% taken as 1th percentile and the data more than 99% is taken as 99th percentile this will remove the outlier data.

result is shown in table 3. As it could be seen, the top two models are BV-GARCH and RV-GARCH Respectively. İt implies that for predicting Gold volatility, the most accurate model is BV-GARCH. İn figure 1 time-series of realized daily variance and conditional variance predicted one-step ahead for the best model, BV-GARCH, is depicted.

Table 3: The Value of Lose Function

MODEL	MSE (*1000000)	Rank	Qlike	Rank
GARCH	6.07505	3	-4084.77	3
EGARCH	6.10756	5	-4073.43	4
GJR-GARCH	6.07724	4	-4051.65	5
RV-GARCH	6.06677	2	-4095.48	2
BV-GARCH	5.88260	1	-4102.83	1



Therefore, we observed that the realized GARCH models (BV-GARCH and RV-GARCH, respectively) had the highest performance in terms of both in-sample and out-of-sample prediction of conditional variance (volatility). Since over-estimation and under-estimation of risk, would costs to the investor, the use of the RGARCH models can minimize this cost.

5. Conclusion

In this paper, using the Gold five-minutes intra-day data from April 2012 to April 2018, we estimate the conditional variance of GARCH, EGARCH, and GJR-GARCH as well as the RGARCH model using two RV and BV proxies for intra-day realized volatility. A good model not only should fit data well but also it should have accurate performance in predicting out of sample volatility. So in this paper both in-sample and out of sample modeling performance has been investigated. We compared models, for in-sample fitting, by the log-likelihood value and used MSE and QLIKE lose functions to evaluate predicting accuracy. The results show that the RGARCH method for GOLD outperforms the other methods in both ways.

Therefore, the use of RGARCH models instead of conventional GARCH models provides a more accurate estimate for the conditional variance as a proxy of volatility which is a key factor in many risk management and portfolio management.

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