Optimal Profit-Loss Sharing Contracts with Symmetric and Asymmetric Information (Principal-Agent Model Approach)

Seyed Hasan Ghavami[a,*]

a. Faculty of Economics, Allameh Tabataba'i University, Tehran, Iran

Abstract

Principal-agent model is one of the most practical models in game theory. This model is used when a person or firm assigns a given work or activity to another person by drawing up a contract. The information of parties might be symmetric or asymmetric. In each case, the performance of the principal-agent model is different. Profit-sharing contracts are essential funding tools in Islamic banking. In profit-sharing contracts, depending on the type of contract the bank concludes with the customer, a separate profit division is determined in the contract. In general, as in the case of the principal-agent model, performing an economic activity is left to the customer. This paper provides an optimal model for profit-sharing contracts based on a principal-agent game theory model under symmetric and asymmetric information and answers these two questions: First, what are the optimal conditions for profit-sharing contracts with symmetric and asymmetric information? Second, does an ideal Islamic society with asymmetric information transform into the one with symmetric information, and does this transformation lead to the elimination of complexity and decrease bank's costs related to the profit-sharing contract? Two hypotheses have been stated to answer the above questions. The first hypothesis is optimal conditions for these contracts, which will be realized by using the achievements of the principal-agent model for profit-sharing contracts. The second hypothesis is that in an ideal society that respects Islamic and Sharia law, completely asymmetric information changes into symmetric one, complexities of the profit-sharing model are eliminated, and bank costs decrease.
Keywords: Asymmetric Information, Islamic Banking, Profit-sharing Contracts, Principal-Agent Model, Symmetric Information.
JEL Classification: C71, D82, D83, G21, G32.

Introduction

Game theory studies cases in which the decision and behavior of one economic actor depends not only on their own behavior and decisions, but also on the behavior and decisions of other players. In that sense, game theory differs from traditional economic theories. In other words, strategic interaction is concerned (Abduli, 2006). When an individual's utility depends not only on their own strategy but also on the strategies chosen by other players', then the strategic behavior will be formed.

One of the applied models in game theory is principal-agent model. This model is a general title for those games in which one player (principal) assigns economic activities to the other player (agent), and motivates them to do the given task, even though there might differences in utility and benefits of players, and they have conflict of interest.

In principal-agent model, information has a fundamental role, and the information of the

---

* Corresponding author email: h.ghavami@atu.ac.ir

principal and agent might be symmetric or asymmetric which could create different conditions for interaction of players. Based on the features of profit-sharing contracts in Islamic economics, these contracts can be analyzed and examined in the framework of principal-agent model. These contracts are used as funding tool in Islamic economics. In these contracts, the bank provides total or part of capital required in economic activities, and participates in profit according to the contract (Bahmanpoor and Moshiri, 2016). This paper seeks to provide a new and comparative method in Islamic economics to be consistent with classical economics.

While explaining the profit-loss sharing contracts by using the principal-agent model of game theory, the present study also seeks to introduce a new method for studying the subjects of Islamic economics. According to that, this paper tries to answer these two questions: What are the optimal conditions of profit-sharing contracts when information is symmetric or asymmetric? And does asymmetric information transform to symmetric one to eliminate complexity, and decrease the bank's costs related to profit-sharing contracts in an ideal Islamic society? The problem has been studied by using the following hypotheses. First, by using achievements and principal-agent model in profit-sharing contracts, the optimum conditions of this kind of contract would be realized. And the second suggests that in an ideal Islamic society being in accordance with Islamic law, asymmetric information would transform to symmetric one leading to elimination of complexity in profit-sharing contracts and decrease in bank's costs. The present study has applied the principal-agent model of game theory, and both hypotheses have been confirmed according to the findings. In addition, this study has shown that even after 14 centuries since the formation of Islamic government, its funding tools are compatible and can be analyzed with today's economic achievements and analytical tools. In addition, by using principal-agent model, a new methodology has been introduced for analyzing profit-sharing contracts in Islamic banking system.

The remainder of the paper is as follows. Section 2 provides a literature review on the principal-agent model, profit-sharing contracts. In section 3, profit-sharing contracts based on principal agent model for two cases of symmetric and asymmetric information are explained. And it concludes with providing the model and results.

Literature Review

Optimum profit-sharing contract with symmetric and asymmetric information (the approach of principal-agent model) is a new topic; however, there are separate studies about principal-agent model and profit- sharing contracts. Reviewing the literature on principal-agent model indicates that this model is rooted in topics which economists have observed inefficiencies of economic interactions, and has sought to resolve. In this regard, Babbage (1835) suggested piece- rates or profit sharing in order to conclude efficient contracts. Barnard (1938) introduced incentive theory in management, and argued that individual's desire for receiving bonus in exchange for their endeavor in profit-sharing system was the key element of organizations. In 1963, Arrow titled that concept as the moral hazard. Wilson (1968) and Ross (1973) expressed it as the Agency problem. Hilman and Laffont (1947) stated the inefficiency of insurance market with asymmetric information, principal-agent model under the moral hazard condition was suggested by economists like, Mirless (1975), Guesnerie and Laffont (1979), Helmstroen (1979), and Grossman and Heart (1983). For the first time, Rogerson Jewitt (1988) and Mireles (1999) used the first-order condition and Lagrange function to analyze scientifically and made principal-agent problem applicable. Pareto inefficiency under moral hazard has been indicated by Rosths child and Stiglitz. Graph analytics of principal-agent model was presented by Scott Birman and Luis Fernandez (1998), Collel et al. (1995), and Laffont and Martimort (2002: 9-19).

There are many studies about principal-agent model under asymmetric information, such as Wahrenburg and Zhou (2002), Feltham (2005), Caillaud (2008).

There have been many studies about the principal contracts. These contracts have been studied from different aspects by a group of researchers, including definition and explanation, from Islamic juridical and legal, Islamic juridical and legal concepts and terms, accuracy and revocation conditions of a contract (Farzinvash et al., 2002).

In addition, some researchers have argued how to use profit-sharing contracts as a tool to pay facilities in Islamic banking (Jamshidi, 2000). Some profit-sharing contracts like bailment have been widely used in Islamic societies throughout the history, and have been studied from different dimensions by Shiite scholars (Mousavi Khomeini, 1989; Najafi, 1988; Yazdi, 2008).

Ismail and Wibow (2015), the scolders of University of Malaysia, asserted that the problem of information in principal-agent model had not much effect on profit-sharing contracts, and by using these contracts, the investment return of the contracting parties would increase.

Ghavami (2017) suggested a theoretical model for adverse selection in Islamic profit-sharing contracts, and showed the conditions which minimized the problem of adverse selection.

Bintimuda et al. (2011) indicated that profit-sharing contracts in Islamic banking would create a value added.

Benamaraoui and Alwardat (2018) showed that how asymmetric information had effect on profit-sharing contract and its costs.

Sugema et al. (2010) argued that profit-sharing contracts were more efficient than interest-based banking contracts.

Sarker (2005) stated the theory of Islamic firm and principal-agent problem in commercial contracts, and showed that Sharia law would minimize the agency problem.

Zahidivafa and Ghavami (2010) suggested a bailment contract model based on principal-agent model, and showed the possibility of bailment contract analysis in the form of principal-agent model.

Abduli and Ghavami (2012) introduced a civil profit-sharing contract in the form of game theory, and comparatively analyzed it with a broad form of games.

In recent years, Chang et al. (2015) considered principal-agent model in which the agent's effort could not be contracted upon, and found that optimal contracts were likely to be "more nonlinear" than in the standard case with concave utility preferences.

Halac et al. (2016) studied a model of long-term contracting for experimentation. They considered a principal agent relationship with adverse selection on the agent's ability, dynamic moral hazard, and private learning about project quality. They found that each of these elements played an essential role in structuring dynamic incentives. Their model permitted an explicit characterization of optimal contracts.

At and Thomas (2017) considered that the revenue from the project was observable and verifiable, but its distribution was influenced by both the borrower's choice of action and the project's quality, which were private information. They found that debt contracts were endogenously optimal, as under moral hazard alone. Moreover, while moral hazard led to credit rationing for the lowest-quality projects only, adding adverse selection created a bang-bang result: either all projects or none are credit rationed.

Barron et al. (2020) characterized the unique profit- maximizing contract, and showed deterring risk-taking affected the insurance-incentive trade-off. Their logic extended to costly risk-taking and to dynamic settings where the agent could shift output over time.

Li and Qiu (2020) investigated the characters that made an optimal contract as well as the conditions under which the utility of a principal and agents could be optimized. As a result,

they showed that, in the pure adverse selection model, the conditions to reach the optimal utility of principal and individual agents were that a principal needed to design different contracts for different types of agents, and an individual agent chose the corresponding type of contracts.

According to the literature review, the subject of this paper is new, and introduces a new method in Islamic economics, and show that first, the optimum conditions of profit-sharing contracts, and second, if Sharia law is implemented in an Islamic society completely (an ideal Islamic society), then the problem of asymmetric information will be solved and subsequently contracts will be efficient, and the their costs will decrease dramatically.

Profit- Loss Sharing Contracts Based on Principal-Agent Model

Principal-agent model is formed based on the gamed theory, and thus has the elements of a game. In principal-agent model, a commitment or duty and activity is assigned to an agent by a principal, and often, the more is the effort of the agent, the more is the profitability of the activity which is desirable to the principal but costly for the agent. In addition, the effort and endeavor of the agent is not observable by the principal and in conclusion of the contract is not considered as a variable to be evaluated.

The agent has usually more information than the principal, and can use this rent information for the benefit of himself. Principal-agent model is analyzed under two distinct conditions including symmetric and asymmetric information and under the later agent has more information than the principal. Due to the complexities of this condition, the model has more constraints. Under asymmetric information, the environment for contract is not transparent and clear, i.e. the one who has more information hide his information or work less than required or gives wrong information to gain more benefit for himself. In principal-agent model with asymmetric information, some constraints are imposed to make the contract environment transparent. This is costly; however, a principal makes the situation at the best by paying the cost.

profit-sharing contracts are funding tools in Islamic economics and in Islamic banking without Reba or interest, contracts are used for granting facilities and profit share of each party is determined as the fraction of joint profit like $\frac{1}{2}, \frac{1}{3}, ...$(Allame' Helli, 1410 Ah) As we seek to provide an optimal model for profit-sharing contracts based on the achievements of principal-agent model it is necessary to clarify under what conditions, a profit-sharing contract is concluded ; in other words, are conditions clear and transparent and information symmetric or they are not clear and profit-sharing contract is concluded under asymmetric information?

In an ideal Islamic society, according to the verses of Qur'an, including, Surah An-Nahl (16:104), Az-Zumar (39:3 & 33), An-Nur (24:7), Al-Hujurat (49:7), Aal-E-Imran (3:17), Al-Ahzab (33:24 and 35), At-Tawbah (10: 119), and in Nahj al-Balaghah (Sermon 86), Mustadrak ul-Wasa'il and Mustanbat al-Masa'il (Vol. 9, pg. 86), the environmental features and the prevailing atmosphere in perfect Islamic societies are without any deception and hypocrisy, so, profit-sharing contracts in banks of ideal Islamic societies are concluded with symmetric information. In addition, according to the Verses of Qur'an including, 17:3, 2:177, Al-Mu'minun (23:8), Al-Ma'idah (5:1), the provisions of the contracts are completely respected and also require enforcement in a perfect Islamic society. Therefore, environment and atmosphere of profit-sharing contracts in societies which are based on Sharia law and behavior and actions of individuals are in accordance with Islamic rules and regulation is symmetric and the problems and complexities of principal-agent model with asymmetric information will be solved. However, banks could be placed in Islamic societies with non-transparent atmosphere and asymmetric information.

Therefore, profit-sharing contracts will be analyzed under two conditions: First, it is assumed that, there an ideal Islamic society in which Sharia law is implemented completely, and people of the society are completely obedient to it and in their interactions they behave in accordance with Islamic rules and consequently, contracts are concluded under symmetric information. In the second situation, it is assumed that some members of the society do not behave according to the Islamic rules and regulation completely and deviation from those rules such as lying, working less than the contractual obligation, and disloyalty to the commitment are observed (asymmetric information).

Therefore, profit-sharing contracts based on the principal-agent achievement will be different in each condition and this paper attempts to offer an optimum model for both conditions. As the model is distinct in each condition, so equilibrium condition leading to the satisfaction and maximum benefit for the bank (the owner) and agent (the customer) is obtained through distinct condition.

Profit- Loss Sharing Contracts based on Principal-Agent Model in an Ideal Islamic Society

In an ideal Islamic society, Islamic rules and regulation are strictly in accordance with Sharia law which governs the society completely, thus, profit-sharing contracts are concluded under symmetric information. Therefore, complexities and the problem of principal-agent model with asymmetric information will be solved and the constraints of the model are reduced to only one constraint which is merely the constraint of profit- loss sharing. In such a case, as in the case of competitive market, solving the model and obtaining the optimal condition of profit- loss sharing contract follow the first-best contract. So the model of profit-sharing contract based on the principal (the bank) agent (the customer) and its optimal conditions are obtained as follows.

Model Assumptions

1. Ideal Islamic conditions governing the contract environment in the society so, the information is perfectly symmetric.
2. The bank, as one of the contracting parties, provides the capital of the profit-sharing contracts and is the principal.
3. The customer demands facilities for economic activities and plays the role of agent.
4. A profit-sharing contract is concluded for gaining profit.
5. The more effort of the agent, with ceteris paribus, the more the profit of economic activities. Which is desirable for the bank (principal), but costly for the customer (agent), and for him this is undesirable.
6. By executing the profit-sharing contract the payoff (total revenue) is determined denoted by x.
7. Agreed costs in the contract (c(x)) are deducted from total revenue. As a result, the profit of profit-sharing contract is obtained in form joint profit and is divided between the bank and the customer with the proportion of (s) and (1-s), according to the agreement.
8. The bank's utility is a function of the bank's share in profit of the contract's execution, as shown by UB S (x-c (x)).
9. The customer's (agent's) utility is a function of his share in Profit, and when his utility is more than other alternatives, he accepts the bank's proposed contract and is shown as the profit-sharing constraint as follows.

$$[u_A(1-s)(x-c(x)) -v(e)] \geq U_A]$$

V(e) indicates the effort function of the agent and as it is undesirable to him is deducted from profit.

-        The bank draws up and offers the profit-sharing contract.

According to the abovementioned assumptions, the bank draws up an optimal contract which its optimal conditions are obtained from the following model.

As the first assumption asserts, the profit-loss sharing contract is used as a funding tool in an ideal Islamic society, therefore the information is symmetric between the bank and the customer, thus the efforts, actions and capabilities of the customer are known and obvious for the bank, and they are shown in a set of A= $\{e_1,...e_n\}$, and each of them has a specific payoff for the executed profit-sharing contract which are indicated in the set of X=$\{x_1...x_m\}$.

According to the abovementioned assumptions, Islamic law governs the society, therefore information is symmetric and individuals choose a contract which coordinates with their condition based on the capabilities. The only condition for the formation of profit-sharing contracts is the provision of minimum utility (U_A) for the customer, the amount that he will get from other alternative opportunities when he does not participate in concluding a profit-sharing contract with the bank. Therefore, as stated by above assumptions, in an ideal Islamic society, optimal model for profit- losssharing contracts, seeks to maximize the bank' utility subject to the agent's participation. Therefore, we have:

1.   Bank's utility function: $U_B = \sum_{j=1}^{n} P(x_j \mid e_i) u_B(s(x_i - c(x_i)))$

2.   Customer's utility function: $U_A = \sum_{j=1}^{n} P(x_j \mid e_i)[U_A(1-s)(x_j - c(x_j)) - v(e_i)]$

In other words, the customer's utility is a function of customer's share, and in form of his net share, which means that the cost of his effort is deducted from the utility. Accordingly, optimal profit-sharing model under an ideal Islamic society is obtained when a bank maximizes the utility function resulting from the contract subject to the constraint of customer's (agent's) participation under symmetric information. Due to the symmetric information, the model has no risk within and only external factors outside the model have effect on its outcomes and cause uncertainty. Now, based on mentioned assumptions and statements, the optimal conditions for the contract in an ideal Islamic society can be obtained by formation a Lagrange function and solving it. The result is a model derived from principal-agent model for profit-sharing contracts.

$$U_B = \max \sum_{j=1}^{n} P(x_j \mid e_i) u_B(s(x_i - c(x_i))) \tag{1}$$

$$\text{s.t}\quad U_A = \sum_{j=1}^{n} P(x_j \mid e_i)[U_A(1-s)(x_j - c(x_j)) - v(e_i)] \geq \underline{U}_A$$

For simplicity, in Lagrange equation it is assumed that $c(x_j) = c_j$.

$$L = \sum_{j=1}^{n} P(x_j \mid e_i) u_B s(x_j - c_j) + \lambda\big[(u_A(1-s)(x_j - c_j) - v(e_i)\big] - \underline{U}_A \tag{2}$$

$$[c_1,...,c_n, \lambda]$$

In fact, Lagrange equation indicates that the utility of bank's profit plus λ equals the utility of agent's profit, which λ is weight factor that determines the minimum utility ($\underline{U}_A$) for the agent. Now by solving the Lagrange equation and by taking derivative with respect to λ and c_j, we have:

$$\frac{dL}{dc_j} = p(x_j le_i)\left[-u'_B s(x_j - c_j) - \lambda u'_A (1-s)(x_j - c_j)\right] = 0 \qquad (3)$$

$$\Rightarrow -\lambda = \frac{su'_B}{u'_A (1-s)}$$

Results show that, the marginal rate of substitution between bank's marginal utility multiplied by bank's share and customer's marginal utility multiplied by his share equals a negative fixed amount for λ in all results. The negative sign for λ means that the bank and customer's (principal-agent) shares move in opposite direction, in other words, as the bank's (principal) share increases the share of customer (agent) decreases.

Notes:

1- The utility in the model is an expected utility and accordingly bank's utility is multiplied by $P(x_j | e_i)$, because despite the fact that customer's effort is known, as there are other factors than e_i being effective on the results, distinct x_j might be obtained.

2- There are two explanation for λ:

a. In classical models of principal-agent model, λ is a fixed number and positive. (Stadler and Castrillo, 1997: 44) while in model of profit-loss sharing contracts it has a fixed amount but negative. That is because in classical models (of principal-agent) the game is a win-lose one, as the agent's utility decreases, the utility of the principal increases. However, in profit-sharing model any increase in benefit and loss of contracting parties move in the same direction and as the profit or loss increases in the profit-sharing contract, the changes in the share of the contracting parties from the profit and loss are equally proportioned. It is worth mentioning that λ does never become zero because in that case, su'_B must be zero or $u'_A(1-s)$ *must equals with infinitive, which are* in contrast with initial assumption for model formation.

b. Lagrange coefficient, λ, estimates the effect of the change in one unit of the constant value of the constraint on the objective function. If λ is positive, then each unit of increase (decrease) in the constant value of the constraint causes a decrease (increase) of about λ in the objective function. If λ is negative, each unit of increase (decrease) in the constant value of the constraint causes an increase (decrease) of about λ in the objective function Edward, T (1994).

c. It is worth noting that in profit-sharing contracts, the profit of an economic activity is based on total revenue minus total costs based on the agreement made in the contract.

Profit- Loss Sharing Contracts based on Principal-Agent Model with asymmetric information

When one of the contracting parties, bank or customer, usually the customer, has more information about his productivity, effort, quality of action and how to make an effort, which is not observable for the bank and cannot be explicitly stated in the contract, and in normal situation could be used by the customer (agent) to the benefit of himself, information is asymmetric and the bank should provide a condition to create incentives for customer to use his information and capability in a way to be beneficial for the bank. In other words, despite of asymmetric information and having no knowledge of using effort and endeavor during the performance of the profit-sharing contracts, bank should draw up a contract to get the maximum effort of the customer for an activity. Thus, in this part, we seek to offer a model that achieves maximum results for profit- loss sharing contracts, under asymmetric information.

Model Assumptions

All assumptions for an ideal Islamic society except the first one, which indicates the symmetric information, hold true for profit-sharing model with asymmetric information, therefore we have:

1. The society is an Islamic one, but not all of its members behave in accordance with Islamic law and in contrast to the ideal Islamic society, those who have more information might use it for the benefit of themselves.
2. Other assumptions are the same as in profit-sharing model in an ideal Islamic society.

It should be noted that in the case of asymmetric information, the number of constraints increase. Thus, this model should contain conditions to give an agent incentive for making effort as required.

Therefore, the optimal amount of effort of the customer (agent) in profit-sharing contracts appears as an incentive constraint as a complementary in the model and we have:

$$e \in \arg \; \underset{\hat{e}}{Max} \left\{ \sum_{j=1}^{n} p(x_j | \hat{e_i})[u_A(1-s)(x_j - C(x_j)) - V(\hat{e})] \right\} \tag{4}$$

where, e refers to effort and $\hat{e}$ is the maximum and optimal effort and the equation indicates the constraint of incentive. This constraint suggests that the agent (customer) uses his maximum effort ($\hat{e}$) when the set of its results maximizes his utility.

Therefore, principal (bank), maximizes its utility subject to the two constraints related to the customer, one is participation constraint and the other is complementary constraint of incentive.

Now, according to the abovementioned description, we can provide a model for drawing up a profit-sharing contract for Islamic banking under asymmetric information and obtain optimal conditions by solving it.

The components of the model include:

1. Bank's utility function: $U_B = \sum_{j=1}^{n} p(x_j | e_i) u_B (s(x_j - c(x_j)))$

2. Customer's (agent's) utility functions which include two constraints, participation and incentive. The constraint of incentive indicates the maximum effort to gain maximum benefit from the contract. Therefore, customer's utility function under asymmetric information is shown as follows.

$$U_A = \sum_{j=1}^{n} p(x_j | e_i) \left[u_A(1-s)(x_j - c(x_j) - V(e_i)) \right] \geq \underline{U}_A . \tag{5}$$

and

$$e \in \arg \; \underset{\hat{e}}{Max} \left\{ \sum_{j=1}^{n} p(x_j | \hat{e}_i)[u_A(1-s)(x_j - C(x_j)) - V(\hat{e})] \right\} \tag{6}$$

For simplicity of the model, we assume $C(x_j) = c_j$ and $e \varepsilon \{e^H, e^L\}$, i.e. we assume that Instead of a wide range of efforts, there are only the high and maximum effort (e^H), or the low and minimum effort (e^L), therefore, the following condition should be drawn up in the contract in order for the agent to make a high and maximum effort.

$$\sum_{j=1}^{n} p(x_j|e^H)[u_A(1-s)(x_j-C_j)-V(e^H)] \geq \sum_{j=1}^{n} p(x_j|e^l)[u_A(1-s)(x_j-C_j)-V(e^L)]$$

$$\Rightarrow \sum_{j=1}^{n} (p(x_j|e^H)-p(x_j|e^L))[u_A(1-s)(x_j-C_j)] \geq V(e^H)-(e^L) \tag{7}$$

Notes:

1. Naturally, for a lazy and idle agent, disunity of high effort is more, so $V(e^H)>V(e^L)$.

2. $p(x_j|e^L)=P_j^L$ and $p(x_j|e^H)=P_j^H$, represent the probability of low effort and high effort by the agent to achieve the result of x_j, respectively.

The principal (bank) prefers high effort to low effort, while in the first choice p_j^H is defeated by p_j^L. Because the agent chooses P_j^L naturally which means that $\sum_{j=1}^{K} p_j^L > \sum_{j=1}^{K} p_j^H$ is true, for all k. In fact, inferior results are more desirable to the agent and naturally, he chooses low effort, thus, $(x_j|e^H)\ p\ (x_j|e^L)$, i.e. expected objective of the contract is high effort and if the goal is low effort, automatically it happens and complementary constraint of incentive is not required. Therefore, profit-sharing model in Islamic banking based on principal-agent model with asymmetric information, is as follows.

$$U_B = \max_{(e,c(.))} \sum_{j=1}^{n} p(x_j|e_i)u_B s(x_j-c_j)\ . \tag{8}$$

s.t

$$\sum_{j=1}^{n} p_j^H[u_A(1-S)(x_j-c_j)-V(e^H)] \geq \underline{U}_A$$

and

$$\sum_{j=1}^{n}(p_j^H-p_j^L)[u_A(1-S)(x_j-c_j) \geq V(e^H)-V(e^L)]$$

Explaining the Model

Equation 8 indicates a model that a bank as a principal maximizes its utility from its share in profit, subject to two constraints related to the customer, participation constraint and complementary constraint of incentive. Comparing this model with profit-sharing contract of an ideal Islamic society indicates that the model becomes more complicated and a new constraint is added, when information is asymmetric in a society which means that more costs are imposed on banks. Therefore if Islamic rules and regulations govern the society and banks, and the members of that society behave in accordance with Islamic law, the profit- loss sharing contract model will become simple and bank's costs, social costs, and transaction cost will decrease. Now, by solving the mentioned model using Lagrange method, optimal conditions of the contract are obtained and based on that, banks could draw up their profit-sharing contracts. To solve by using Lagrange method, we have:

$$L = \sum_{j=1}^{n} p_j^H \left[u_B s(x_j-c_j)\right] + \lambda \left[\sum_{j=1}^{n} p_j^H [u_A(1-s)(x_j-c_j)-V(e^H)-\underline{U}_A]\right] + \mu \left[\sum_{j=1}^{n}(p_j^H-p_j^L)[u_A(1-s)(x_j-c_j)-V(e^H)+V(e^L)]\right]$$

$$\left[C(x_j),\lambda,\mu\right] \tag{9}$$

By taking derivative[1] of Equation 9 with respect to (c_j) and using first order condition, we have:

$$\frac{su'_B}{u'_A(1-s)} = -\lambda - \mu(1 - \frac{P^L_j}{p^H_j}) \tag{10}$$

Analyzing the Results of the Model

First, according to Equations 3 and 10 the first hypothesis is and by comparing these two, the second hypothesis is confirmed.

Second, by solving the Lagrange function we will have $\mu \neq 0$. if $\mu = 0$, then $p^L_j = p^H_j$, i.e. the problem is for conditions with symmetric information. In that case, the complementary constraint of incentive will not exist, therefore a fixed share in participation leads to minimum effort and $-\mu > 0$ requires $p^L_j > p^H_j$. This is consistent with Kuhn-Tucker condition and first condition and requires a positive and limitative shadow price which means that with asymmetric information, profit-sharing contracts impose more costs on banks. However, with symmetric information about the level of effort, bank's profit is more than the case with asymmetric information. Therefore, in Islamic banking, optimal profit-loss sharing contracts are more profitable in an ideal Islamic society than an Islamic society where, Islamic law is not fully respected and information is asymmetric and information becomes symmetric when $p^L_j < p^H_j$ thus we have $-\mu < 0$ which is not consistent with Kuhn-Tucker condition and first condition.

As, $\mu \neq 0$, there is moral hazard due to asymmetric information. Agent's share in profit changes according to the results, and the less the $\frac{P^L_j}{p^H_j}$, the more the agent's share in profit.

In other words, any decrease in $\frac{P^L_j}{p^H_j}$, increase the probability of high effort (e^H) In order to encourage agent to make high effort, more profit should be paid to him. Therefore, with asymmetric information, payments to the agent (agent's share in profit) depends on his effort, and as the payments of profit increase, the agent's effort increase, as well.

Third, comparing the results of profit-sharing contract with that of principal-agent model in classical economics, shows that the results are completely different but the formulation is the same. In principal-agent model, the ratio of marginal utility of principal to the agent equals a positive fixed value (λ) plus a fixed coefficient (μ) (Stadler and Castrillo (1997, p.44). It means that, as the agent's utility decreases, bank's utility increases proportionally. μ and λ, estimate the effect of one-unit change in fixed value of objective function. μ and λ are positive in conventional principal-agent models, which means that, any increase in fixed

---

1. Remember that, $\sum\limits_{j=1}^{n} p^H_j = \sum\limits_{j=1}^{n} p^L_j = 1$

$$\frac{\partial L}{\partial c_j} = -P^H_j su'_B - \lambda P^H_j u'_A(1-S)(x_j - c_j) - \mu(P^H_j - P^L_j)u'_A(1-S)(x_j - c_j) = 0$$

$$\Rightarrow \frac{p^H_j su'_B}{u'_A(1-S)(x_j - c_j)} = -\lambda P^H_j - \mu(P^H_j - P^L_j)$$

$$\Rightarrow \frac{u'_B}{u'_A(1-S)(x_j - c_j)} = -\lambda - \mu(1 - \frac{P^L_j}{P^H_j})$$

values of the constraint function leads to decrease of about λ in objective function. But μ and λ are negative in profit- loss sharing model, so, any increase in fixed value of constraint functions lead to increase in objective function by about μ and λ. The other fundamental difference of profit- loss sharing contract based on principal-agent model with conventional principal-agent model is that, in the later the ratio of marginal utility of the bank to the agent's marginal utility equals to positive coefficients of λ and μ. Whereas in profit-sharing contracts, the ratio of bank's marginal utility multiplied by its contractual share to the agent's marginal utility multiplied by his contractual share equals to fixed negative coefficients of λ and μ which reflects positive effect of constraint functions on objective function and indicates that profit and loss of contracting parties move in the same direction. The reason for this is that, in classical models of principal - agent, as agent's utility decreases, the principal's utility increases proportionally, i.e. as the game is win-lose, the less the agent's share in total outcome (the payoff of the game), the more the share of principal in payoff of the game, proportionally. However, in profit- loss sharing models of Islamic banking, any increase in profit and loss of contracting parties move in the same direction. In profit-loss sharing contracts, when profit or loss increases, the share of contracting parties from profit and loss changes, respectably, i.e. any change in payoff of the game in profit-loss sharing contract has direct effect on both parties with respect to their share. However, in conventional model of principal-agent in a contract, since the profit division is based on a zero-sum game, as agent's profit decreases, principal's profit increases respectably. While in profit-sharing contract model, profit and loss are divided in proportion to the contractual share between the principal (bank) and the agent (customer). Given that, any increase in profit or loss leads to increase in contracting parties' (agent and principal) profit or loss. Thus, in profit-sharing contracts of Islamic banking the division of profit follows a cooperative game.

Briefly we compare the characteristics and optimization conditions of Profit-loss Sharing contracts and conventional classical economy contract in the form of principal–agent model in Table 1.

Table 1: Comparison of Islamic Profit-loss Sharing Contracts based on Principal–agent Model with the Contracts in the form of Principal–agent Model in Classical Economics

Islamic Profit-loss Sharing Contracts based on principal–agent model	Contracts in the form of principal–agent model in classical economics
Assignment of economic activity to agent based on share of profit	Assignment of economic activities to agent based on definite wage
Optimal condition of contract in ideal Islamic society (symmetric information) $-\lambda = \dfrac{su'_B}{u'_A(1-s)}$	Optimal condition of contract in the case of symmetric information $= \lambda = \dfrac{1}{u'A(w)}$
Optimal condition of contract in non-ideal Islamic society (asymmetric information) $\dfrac{su'_B}{u'_A(1-s)} = -\lambda - \mu(1 - \dfrac{P^L_j}{p^H_j})$	Optimal condition of contract in the case of asymmetric information $\dfrac{1}{u'A(w)} = \lambda + \mu\,(1 - \dfrac{p^l_j}{p^h_j})$
The **Lagrangian** coefficient sign is negative in Islamic **partnership** contracts.	The Lagrangian coefficient sign is positive in classical economy contract.

Source: Research finding.

According to Table 1:
1. In partnership contracts (Islamic model) based on principal–agent model, since the contracts are based on shareholding, λ is equivalent to the principal's profit share

multiplied by the marginal utility of the agent's profit share, while in the classical economics, the marginal utility of agent's wage determines the optimal condition.

2. The Lagrangian coefficient in the Islamic model is negative. This does not mean that λ and μ are negative in the Islamic model. Rather, it indicates that the principal and agents' share of profit also change direction. That is, if the principal's profit decrease with the execution of the contract, the agent's profit also decrease and vice versa. This is not the case in the conventional   principal–agent model in the classic economy. No matter how much the principal's profit from the execution of the contract increases, the agent's wage does not change. Therefore, in this case, the ratio of agent's profit to principal's profit reduces.

References

[1] Abduli, G. (2007). *Game Theory and Its Applications, Static and Dynamic Games with Perfect Information.* Tehran: Jihad-e- Daneshgahi Publishing Organization (In Persian).

[2] Allame Helli (1410 AH). *Rules of Rulings*. Al-Yanabi 'al-Fiqhiyah Dynasty, Beirut: Islamic Dar al-Islamiyyah (In Persian).

[3] Ali Ibn Abi Talib (AS) (2010). *Nahj al-Balagha*, Qom: Al-Hadi Publishing Office (In Persian).

[4] Al-Najafi, S. M. H. (1981). *Jewel of the Word*. Beirut: Al-Ahya Al-Tarath Al-Arabi (In Persian).

[5] At, C., & Thomas, L. (2017). Optimal Lending Contracts. *Oxford Economic Papers*, *69*(1), 263–277.

[6] Barron, D., Georgiadis, G. &, Swinkels, J. (2020). Optimal Contracts with a Risk-Taking Agent. *Theoretical Economics*, *15*(2020), 715–761.

[7] Chang, H., Cvitanic, J., & Yyuzhou, X. (2015). Optimal Contracting with Moral Hazard and Behavioral Preferences. *Journal of Mathematical Analysis and Applications*, *428*(2), 959–981.

[8] Farzinvash, A. (2002). *Studying Contracts in Islamic Banking*. Tehran: Monetary and Banking Research Institute, Central Bank of the Islamic Republic of Iran (In Persian).

[9] Ghavami, S. H. (2017). Theoretical Model for Adverse Selection Problem in Islamic Banking Participation Contracts. *Eghtesad-E Islami Jounal*, 17(66), 191–214 (In Persian).

[10] Halac, M., Kartik, N., & Liu, Q. (2016). Optimal Contracts for Experimentation. *Review of Economic Studies*, *83*(3), 1040–1091.

[11] Ismail, A. G., & Wibwo, A. (2015). Optimal Profit Sharing Contract and Principal–Agent Value in Islamic Bank University Utara, Malaysia. *IMF Working Paper*, Retrieved from
www.researchgate.net/publication

[12] Jamshidi, S. (2000). *Islamic Banking, Equipping and Allocating Resources, Methods for Granting Accounting Facilities*. Tehran: Shokooh Andisheh Publications (In Persian).

[13] Laffont, J. J., & Martimort, D. (2002). *The Theory of Incentives the Principal – Agent Model*. New Jersey: Princeton University U.K.

[14] Li, C., & Qiu, Z. (2020). Optimal Contracts for Agents with Adverse Selection. *Discrete Dynamics in Nature and Society*, *2020*, 1–17.

[15] Mousavi Khomeini, R. (1989). *Tahrir al-Waseela*. Tehran: Islamic Publications Office affiliated with the Qom Seminary Teachers Association (In Persian).

[16] Sarker, A. A. (2005). Islamic Business Contracts, Agency Problem and the Theory of the Islamic Firm. *International Journal of Islamic Financial Services, 1*(2), 1-15.

[17] Sugema, I., Bakhtiar, T., & Effendi, J. (2010). Interest versus Profit-loss Sharing Credit Contract: Efficiency and Welfare Implication. *International Research Journal of Finance and Economics, 45*, 1450–2887.

[18] *The Holy Qur'an.*

[19] Zahidivafa, M. H., & Ghavami, S. H. (2010). Mudaraba's Model Based on the CEO's Model under the Moral Hazard Condition. *Economic Research Journal*, *11*(4), 239–255 (In Persian).