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RESEARCH PAPER

Optimum Public Debt in Iran: An Emphasis on The Precautionary Savings Model

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Abstract
This study examines the effects of the government debt-to-GDP ratio on welfare in the Iranian economy. Because of limited access to loans in Iran, the analysis employs an incomplete-markets model based on Aiyagari's heterogeneous-agent framework. In addition to idiosyncratic wage risk, households face both budget and borrowing constraints. Given that the Iran's economy faces both high inflation and borrowing constraints, Iranian households prefer to hold assets such as gold, land, and term deposits, besides government debt and physical capital, for precautionary savings purposes. Therefore, this study incorporates the above-mentioned assets in the model. The model is solved numerically using the finite element method (FEM). The results show that the optimal quantity of debt is significantly negative according to the welfare criteria, and the welfare gain from moving to the optimum is 2.66% of per capita consumption, which is not negligible. As a result, the negative role that debt plays in the model (crowding out private capital) is more than the positive role of enhancing liquidity, and private sector spending should replace government spending. It is optimal for the government to accumulate assets in the long run so that all government expenditures are financed by the interest income earned on private assets. Furthermore, if excessive holding of assets is restricted through policies such as imposing taxes on holding assets and developing financial markets, the welfare loss of maintaining the current level of debt instead of the optimal quantity will decrease.
Keywords: finite element method, government debt, heterogeneous agent models, incomplete markets, precautionary saving, self-insurance.
JEL Classification: C61, G12, G51, H63.

Introduction

The notion of "fiscal sustainability" pertains to fluctuations in public debt. The debt rule, introduced by the International Monetary Fund (IMF) in 2009 as the debt-to-GDP ratio, establishes a benchmark or ceiling on government debt as a share of GDP. It is widely regarded as a crucial determinant of economic well-being and a core element of government fiscal sustainability (Kordbacheh, 2018: 25).

According to 2021 statistics, the highest government debt[1] ratio was 262.5% (Japan), followed by Venezuela at 241%. For Iran the ratio stood at 41.5%. Moreover, the IMF reported central government debt-to-GDP ratios for Iran of 39.5% in 2020 and 42.4% in 2021[2].

Several studies have calculated the optimal debt ratio on the basis of its welfare effects, employing models characterized by heterogeneous agents and incomplete markets. Household saving behavior is shaped by precautionary motives and borrowing constraints. In such models, individuals usually face income fluctuations due to idiosyncratic shocks. The environment of complete markets and so-called Arrow–Debreu or Debreu models provide full access to insurance markets, including lending and borrowing markets for households to smooth their consumption. Moreover, agents fully insure themselves against income risks through insurance markets (i.e., external insurance or insurance by another). But in incomplete market models, the loan and insurance markets are assumed to be eliminated, so the only option for individuals to deal with income fluctuations is self-insurance by holding a single risk-free asset such as fiat money, credit (including inside money, government bonds, private IOUs, bank deposits, etc.), and capital.

In the studies conducted in the framework of precautionary savings models, government debts play a different role compared to the standard representative-agent growth model. In other words, government debt appears in the form of an asset as precautionary savings, reducing borrowing constraints and smoothing consumption over time. Besides, government debt causes capital to be crowded out, which then decreases production and consumption (Aiyagari and McGrattan, 1994: 448, 465).

Quantitative analysis by Aiyagari and McGrattan (1994) showed that the effects of crowding out for welfare are unavoidable and the welfare benefit of reducing the current level of debt to its optimal level is high in US (i.e., about 4% of consumption). In a similar study, Aiyagari and McGrattan (1998) showed that the optimal level of debt is not different from the current level in US, and the welfare benefits of being at the optimal level of debt instead of the current level are insignificant. Flodén (2001) also found in the context of such models that if transfer payments are less than the optimal level, debt will increase welfare. But,

[1]. It may indicate general government debt.
[2]. Retrieved from https://tradingeconomics.com/.

if transfers are optimal, the positive effects of public debt will disappear. Indeed, transfer payments usually reduce inequality by changing the distribution of resources between households. However, the taxes required to finance these transfers distort labor supply and savings. He calculated the welfare gains of moving from the benchmark debt ratio to the optimal ratio, which will be 3.4% of consumption. Nakajima and Takahashi (2017) also analyzed the effect of government debt on welfare. They showed the welfare loss of maintaining current level of debt instead of the optimal quantity is 0.19% of consumption in Japan. They revealed that increasing government assets would decrease taxes and improve welfare. By decomposing the welfare gain obtained by increasing government, they found that most of the welfare gain arises from an increase in the average level of consumption instead of reductions in the uncertainty and inequality faced by households (as quoted by Nakajima and Takahashi, 2022).

Overall, the importance of the debt issue is undeniable. Based on the latest information derived from internal reports[1], Iran's government liabilities for interest and principal payments on government securities have resulted in an increase in the ratio of government and government-owned corporation debt to GDP and the ratio of the need for gross government financing to GDP surpassing the sensitivity threshold and limits specified in Article 8 of the Sixth Development Plan Law of Iran[2]. As a result, the government budget is becoming increasingly reliant on financing via the issuance of bonds. The study by Fathalizadeh (2016) also indicates financial instability in Iran. Yet few studies have examined the Iranian economy in this context. Therefore, the purpose of the present study is to address the issue of debt in this regard and calculate its optimal quantity for the Iranian economy. As Khosravi, Azarbayjani, and Nadri (2018) pointed out, limited access to loans is one of the factors affecting high savings in Iran, and this type of savings is referred to as precautionary savings. In the study by Hoseiny et al. (2018), the incompleteness of markets in Iran has been mentioned due to the limited credit markets and the lack of unemployment insurance coverage. Therefore, the borrowing constraint in Iran should be considered a serious limitation in household behaviour. Using household budget data, Einian (2015) has also shown that from

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[1]. For 2015, 2016, and 2017, the Government Assets and Liabilities Management Center of Iran computed the government sector debt ratios as 39.0, 44.6, and 35.7% of GDP, respectively. The ratio is estimated by the Parliament Research Center of Iran to be approximately 48 and 53% for the years 2019 and 2020, respectively, based on data from the Central Bank of Iran and about 34 and 37% based on the Statistical Centre of Iran, respectively.

[2]. Maintaining the government and government-owned corporations' debt ratio to GDP at a maximum level of 40%.

1985 to 2015, more than 20% of Iranian households have never received any loan. According to this evidence, despite the high growth of the money supply (M2), Iran's economy is limited in borrowing. Hence, the focus of this study is on the government debt and the incompleteness of markets in the real world, to examine more precisely the effects of debt on the economy. Therefore, the outcome of its impact in the framework of precautionary savings models, considering the specific conditions of Iran's economy, has been investigated using the finite element method (FEM)[1]. As a result, the model is calibrated, and based on evaluating the welfare effects of debt, the optimal quantity of government debt has been examined in different cases.

The remainder of this paper is structured as follows. The next section describes the literature review. The model is presented in the section 3. Section 4 calibrates the model to the Iranian economy. The results and the robustness of the results are presented in Sections 5 and 6, respectively. Finally, concluding remarks are presented in Section 7.

Literature Review

According to the Keynesian view, as consumers are myopic and base their decisions on current income, they perceive government debt as a form of net wealth and increase their consumption when government taxes are reduced. In the short run, it raises demand for goods and services, leading to corresponding increases in production and employment levels. However, in the long run, as investors compete for a smaller pool of savings, interest rates increase, which in turn discourages investment. Over time, the stock of capital goods and national production are both negatively affected by the reduction in national saving caused by tax cuts (Abbasian and Nouri, 2007). The consequences of government debt are not therefore explicable through a linear trend. Indeed, in this view, government debt primarily has a positive impact on economic growth to some extent and subsequently it has a negative effect. These effects are explained by the Laffer curve (Mousavinik and Bagheripormehr, 2019).

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[1]. Solving a functional equation is an essential step in many economic problems. Typically, the goal is to find decision functions that satisfy Euler's conditions or a value function that satisfies Bellman's equation. However, in many cases (such as nonlinear problems), it is not possible to derive analytical solutions for these functions, and it is required to use numerical methods, such as weighted residual and finite element methods (McGrattan, 1999). In addition, McGrattan (1996) showed that the algorithm for solving functional equations is fast and accurate, whereas the iterations of the value function require much more time.

Barro (1974) argues that the private sector's valuation of government bonds as net wealth is essential for illustrating the consequences of fluctuations in the public debt stock. An increase in government debt will raise net wealth in the presence of imperfect private capital markets if either the government intermediates loans more efficiently than the private market or it has monopoly power to provide "liquidity services" of bonds. In contrast, with respect to the tax obligations and risk associated with debt financing and government debt, an increase in the stock of government bonds may augment the overall risk of the household balance sheet, consequently leading to a decline in household wealth (Barro, 1974). Thus, there is disagreement among various economic schools about the positive impact of debt or its negative impact, and there is even debate among debt proponents about its extent. It is noteworthy that there is no set threshold for debt, and it will alter as economic, social, and political conditions change. Also, the combination of debts and bond buyers can affect this threshold. Therefore, several factors including economic growth, operational plan, and debt buyer (domestic or foreign) affect the vulnerability of any economy to the amount of debt (Ministry of Economic Affairs and Finance, 2016).

In recent years, there has been a growing body of literature that seeks to re-examine significant macroeconomic questions using models with heterogeneous agents. Models in the literature commonly assume the presence of idiosyncratic shocks to individuals' income, along with incomplete markets and borrowing constraints. Therefore, some studies in the economic literature have calculated the optimal debt ratio based on the welfare effect of government debt and have used models in which the analyzed space has these two characteristics (heterogeneous agents and incomplete markets). Aiyagari (1994) showed that in an economy where there are borrowing constraints, meaning the market does not allow households to borrow as much as they need, precautionary saving will result. Household saving behavior is affected by precautionary saving and borrowing constraints. Additionally, in such models, individuals face income fluctuations due to "idiosyncratic shocks". The term "idiosyncratic risk" refers to a risk that does not affect the overall distribution of assets and only leads to the reshuffling of individuals within the asset distribution. Thus, agents are heterogeneous in terms of income and consumption processes. In contrast to representative-agent models, in which all individuals experience the same income realization and therefore the same consumption path and are homogeneous in this respect.

The environment of complete markets and so-called Arrow–Debreu or Debreu models provide full access to insurance markets, including lending and borrowing markets for households to smooth their consumption. Moreover, agents fully insure themselves against income risks through insurance markets (external insurance or insurance by another). But in incomplete market models, the loan and insurance markets are assumed to be absent, so the only option for individuals to deal with income fluctuations is self-insurance by holding a single risk-free asset such as fiat money, credit (including inside money, government bonds, private IOUs, bank deposits, etc.), and capital. According to Huggett and Ospina (2000), the “excessive” accumulation of assets can be considered as aggregate precautionary saving of the economy (Ljungqvist and Sargent, 2018: 820), and such models are called Bewley models (Hosseini, 2013: 225). In Bewley-type models, the income of all households is considered the same at first. Then, over time, according to the different shocks that each household perceives, the wealth and income of individuals will differ.

There are several Bewley models in which a continuum of agents has an identical saving problem, and their behavior produces an asset demand function. The models are (1) Aiyagari’s (1994; 1995) model: the asset is physical capital or private IOUs, and the net supply of the asset is available with physical capital; (2) Huggett’s model (1993): the asset is private IOUs, and available in zero net supply; (3) Bewley’s model of fiat currency; (4) modified Bewley’s model: permitting an inflation tax; and (5) modified Bewley’s model: paying interest on currency through deflation (Ljungqvist and Sargent, 2018: 806).

In the studies conducted in the framework of precautionary saving models, models in which government debt plays a different role compared to the standard representative-agent growth model take into account the cost-benefit trade-offs. In other words, government debt appears in the form of an asset as precautionary saving, and increases household liquidity, reducing borrowing constraints and smoothing consumption over time (liquidity-enhancing effect). Conversely, government debt causes capital to be crowded out, which then decreases production and consumption (capital crowding-out effect). Based on this approach, if debt smooths consumption over an agent's lifetime, the optimal level of debt will be high. If it crowds out capital, thereby reducing consumption, it will be low (Aiyagari and McGrattan, 1994: 448, 465). Studies have been conducted on the optimal quantity of debt in Iran and other countries, some of which are mentioned in Table 1.

Table 1. Studies Conducted on the Optimal Debt Level in Iran and Other Countries

Author & Year	Title	Approach	Risk-Free Assets	Main Findings/Outcome
Aiyagari and McGrattan (1994)	The Optimum Quantity of Debt	A growth model with uninsured idiosyncratic shocks [an augmented version of the model in Aiyagari (1994)]	Capital and Government Debt	The effects of crowding out for welfare are unavoidable. The optimal level of the US government debt is approximately −2.5 times GNP, and the welfare benefit of reducing the current level of debt to its optimal level is high (about 4% of consumption).
Aiyagari and McGrattan (1998)	The Optimum Quantity of Debt	A growth model with uninsured idiosyncratic shocks [an augmented version of the model in Aiyagari (1994)]	Capital and Government Debt	The optimal level of debt is not different from the current level in the United States, and the welfare benefits of being at the optimal level of debt instead of the current level are insignificant.
Flodén (2001)	The Effectiveness of Government Debt and Transfers as Insurance	An Aiyagari (1994)-style heterogeneous agent and incomplete markets model	Capital and Government Debt	If transfer payments are less than the optimal level, debt will increase welfare. The optimal quantity of debt is −100% of GDP in the US. The welfare gains of moving from the benchmark debt ratio (67% of GDP) to the optimal ratio (−100%), is 3.4% of consumption.
Nakajima and Takahashi (2017)	The Optimum Quantity of Debt for Japan	An Aiyagari (1994)-style heterogeneous agent and incomplete markets model	Capital and Government Debt	The optimum government debt ratio is −0.5 in Japan. Welfare loss of maintaining the current level of debt (130%) instead of the optimal quantity is 0.19% of consumption in Japan. Increasing government assets decrease taxes and improve welfare.
Mousavinik and Bageripormehr (2019)	Constructing Government Time Series Debts and Estimating the Optimal Ratio of Government Debt to Gross Domestic Product and Financial Space in Iran Economy	The Smooth Transition Regression Approach	–	The extracted optimal government's debt ratio is 19 percent. The calculated financial space based on two ways (the highest government debt ratio has been experienced and the ratio of government debt that leads to negative economic growth) shows the government's financial space to create debt will be around 30 to 32% of the debt-to-GDP ratio.
Okamoto (2022)	The Optimum Quantity of Debt for an Aging Japan: Welfare and Demographic Dynamics	Extended Lifecycle General Equilibrium Model with Endogenous Fertility	–	Japan's optimal net government debt to GDP ratio depends on the two viewpoints. From the efficiency viewpoint, it is negative, with a ratio of −170%, whereas from the future population viewpoint, it is positive, with a ratio of 220% (or 230%) approximately from 2045 to 2150. Since, one policy instrument cannot achieve two policy goals, it may be better to improve per-capita welfare by reducing debt ratio.

As observed, the studies that have used the Aiyagari (1994)-style model to calculate the optimal level of debt have considered capital and government debt as risk-free assets. However, in contrast to other countries in the world, due to the high inflation in Iran's economy and the lack of taxation on assets, Iranian households prefer to hold some assets such as gold, land, and term deposits for precautionary saving. Therefore, these assets are also included as risk-free assets in the model, in addition to capital and government debt.

Model

Since there is no set threshold for debt, and it will change as economic, social, and political conditions change, it is necessary to choose an appropriate model to examine this issue for the economic conditions of Iran. As mentioned, according to studies conducted by Khosravi, Azarbayjani, and Nadri (2018), Hoseiny et al. (2018), and Einian (2015) in Iran's economy, despite the high growth of money supply (M2), households face limited access to loans, which is one of the effective factors in high saving in Iran. This constraint is referred to as borrowing constraint in economic literature, and the resulting saving is known as precautionary saving.

Therefore, the analysis presented here is based on an Aiyagari (1994)-style heterogeneous agent and incomplete markets model. Since such a model is a general equilibrium version of Bewley's model, the model is also called the Bewley model or Bewley-Aiyagari model. Households face idiosyncratic wage risk and are budget- and borrowing-constrained. Labor supply is endogenous. Some researchers (e.g., Aiyagari and McGrattan, 1994; 1998) using the same framework, analyzed the welfare effects of government debt in the United States, Japan, etc. After choosing a model in accordance with the prevailing conditions in Iran's economy, we extended their models by incorporating other assets (in addition to capital and debt) that Iranian households use for precautionary saving, including precious metals (gold and gold coins), land, and term deposits. Regarding the treatment of debt as net wealth, studies examining the validity of the Ricardian equivalence hypothesis in Iran's economy have been reviewed. With the exception of those by Samadi (2006) and Hafezi and Amir Yousefi (2008), the research conducted by Abbasian and Nouri (2007), Farzinvash and Farahbakhsh (2011), Zamanzadeh et al. (2013), Jafari Yansari (2015), Jeyhoontabar (2016), Sadeghi Shahedani et al. (2020), and Goudarzi Farahani et al. (2020) indicates rejection of this hypothesis in Iran. This means that consumers perceive government debt as net wealth and, by increasing it, enhance their consumption.

As Ljungqvist and Sargent (2018) have pointed out, the tools for formulating the models are: (1) using discrete-state discounted dynamic programming, to construct and solve problems of the individuals, and (2) using Markov chains to compute an invariant wealth distribution. The models produce an invariant wealth distribution that is determined concurrently with various aggregates that are defined as means across corresponding individual-level variables.

Firms

A representative firm hires two factors of production, physical capital and labor, to produce a single good. The neoclassical aggregate production function is:

$$Y_t = F(K_t, z_t N_t) \tag{1}$$

$$Y_t = {K_t}^{\theta}(z_t N_t)^{1-\theta} \tag{2}$$

where Y_t is per capita output, K_t is per capita capital stock, N_t is per capita effective labor force, $\theta \in (0,1)$ is the capital's share of output, and z_t is labor-augmenting productivity in the time period *t*. Productivity grows at a constant rate of g or $z_t = z(1+g)^t$; assuming the product and factor markets are competitive, the wage rate w_t and the interest rate r_t are given by:

$$r_t = F_1(K_t, z_t N) - \delta \tag{3}$$

$$w_t = z_t F_2(K_t, z_t N) \tag{4}$$

then,

$$r_t = \theta {K_t}^{\theta-1}(z_t N)^{1-\theta} - \delta = \theta \frac{Y_t}{K_t} - \delta \tag{5}$$

$$w_t = (1-\theta){z_t}^{1-\theta}{K_t}^{\theta}N^{-\theta} = (1-\theta)\frac{Y_t}{N} \tag{6}$$

where $\delta \in (0,1)$ is the capital depreciation rate. A proportional tax imposed on the labor and interest income (on assets) is denoted by τ_y. The after-tax wage $\bar{w}_t$, and the interest rate $\bar{r}$ are given by:

$$\bar{r} = (1-\tau_y)(F_1(K_t, z_t N) - \delta) = (1-\tau_y)r \tag{7}$$

$$\bar{w}_t = (1-\tau_y)z_t F_2(K_t, z_t N) = (1-\tau_y)w_t \tag{8}$$

Households

There is a continuum of infinitely-lived households, and the total measure of agents is normalized to one. They face idiosyncratic shocks to their labor productivities (which differ in their labor productivity), and they are in each period subject to idiosyncratic productivity shocks. Indeed, agents are ex-ante identical but ex-post heterogeneous due to a random income process. An individual's labor productivity is denoted by e_t, and it is assumed that it is i.i.d[1] across agents. The productivity process is a finite state Markov chain, $e \in \{e_1, \dots, e_m\}$. To obtain the Markov chain, an AR(1) process, $\ln e_t = \rho \ln e_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, is approximated by Tauchen (1986)'s method (due to estimated persistence of idiosyncratic productivity)[2]. The income process is normalized to unity so that $E(e_t) = 1$. The transition probability from state i to state j is given by $\pi_{i,j}, i, j = 1, \dots, m$. All households face identical productivity stochastic process. It is also assumed that there are no aggregate shocks.

Let c_t, l_t, a_t denoting an individual's consumption and leisure time in the period *t*, and an individual's assets at the origin of the period *t*, respectively. The time endowment for the household is normalized to 1. Households are identical in their preferences, and the instantaneous utility function is specified as $\frac{(c_t^\eta l_t^{1-\eta})^{(1-\mu)}}{(1-\mu)}$, where μ is the relative risk aversion coefficient, and η is the preferences parameter (the share of consumption). The a_0 and e_0 are assets and productivity shock at time t_0. The consumer, to maximize the discounted utilities subject to the constraints, chooses stochastic processes for consumption, labor hours, and asset holdings. Agents are budget- and borrowing-constrained (that is, net financial wealth cannot be negative, $a_t \geq 0$) and do not have access to private insurance markets. Therefore, asset markets are incomplete, and due to the precautionary motives and borrowing constraints, the consumer will hold assets to buffer earnings shocks. According to the specific conditions of Iran's economy, there are five risk-free assets, including physical capital (K_t), government debt (B_t), precious metals ($PRMTL_t$), land ($LAND_t$), and term deposits (TD_t) in the model and households use savings to self-insure against idiosyncratic wage risk. Then, A_t denotes per

[1]. Independent and Identically Distributed

[2]. The finite-state Markov chain approximation methods developed by Tauchen (1986) and Tauchen and Hussey (1991) are widely used in solving functional equations where the state variables follow AR processes. The set of integral equations derived from nonlinear dynamic macroeconomics and asset pricing models often does not admit explicit solutions. Therefore, an effective tool to reduce the complexity of the problem is discrete-valued approximations. The Markov-chain approximation methods choose discrete values for the state variables and make transition probabilities such that the characteristics of the generated process mimic those of the underlying processes. However, for highly persistent autoregressive (AR) processes or processes with characteristic roots close to unity, both Tauchen (1986) and Tauchen and Hussey (1991) pointed out that these methods do not work well. Although that type of approach can generate a better approximation with a finer state space, it is not always feasible. Given the prevalence of highly persistent shocks in dynamic macroeconomic models, a new research interest was sparked, and Rouwenhorst (1995) proposed a Markov-chain approximation of an AR(1) process constructed by targeting its first two conditional moments for strongly autocorrelated processes (as cited in Gospodinov and Lkhagvasuren, 2011).

capita assets held by consumers, and indeed, the asset market equilibrium condition is defined by:

$$A_t = K_t + B_t + PRMTL_t + LAND_t + TD_t \tag{9}$$

Also, the labor market clear condition is defined as:

$$N = Ee_t(1 - \ell_t)$$

The optimization problem for households is written follows:

$$\max_{\{c_t,\ell_t,a_{t+1}\}} E\left[\sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\eta}\ell_t^{1-\eta})^{(1-\mu)}}{(1-\mu)} |a_0, e_0\right] \tag{10}$$

$$\text{subject to} \quad c_t + a_{t+1} \leq (1+r)a_t + w_t e_t(1-\ell_t) + TR_t - T_t \tag{11}$$

$$c_t \geq 0, 0 \leq \ell_t \leq 1, a_t \geq 0, t \geq 0 \tag{12}$$

where TR_t is transfer payments, and T_t is tax.

Government

The government issues debt and imposes taxes on labor and interest income (on assets). Tax revenues are allocated to government expenditures, transfers, and interest payments on the debt. The government budget constraint is defined as:

$$G_t + TR_t + rB_t = B_{t+1} - B_t + \tau_y(w_t N + rA_t) \tag{13}$$

and by substituting (9):

$$G_t + TR_t + rB_t = B_{t+1} - B_t + \tau_y[w_t N + r(K_t + B_t + PRMTL_t + LAND_t + TD_t)] \tag{14}$$

where, G_t is per capita government expenditures, and B_t is per capita government debt.

The Stationary Form of The Model

It is necessary to convert the model to a stationary (non-growing) form by dividing equations by Y_t. For this purpose, let $k = \frac{K_t}{Y_t}$, $\widetilde{w} = \frac{w_t}{Y_t}$, $\tilde{c} = \frac{c_t}{Y_t}$, $\tilde{a} = \frac{a_t}{Y_t}$, $\gamma = \frac{G_t}{Y_t}$, $b = \frac{B_t}{Y_t}$, $prmtl = \frac{PM_t}{Y_t}$, $land = \frac{LAND_t}{Y_t}$, $td = \frac{TD_t}{Y_t}$, $\bar{a} = \frac{A_t}{Y_t}$.

In the analysis, it is focused on an equilibrium where Y_t, T_t, B_t, and A_t grow at the rate of g. Then, the stationary form of the model is rewritten as follows.

The optimization problem of households after being divided by Y_t (i.e., stationary form) and applying a proportional tax takes the following form:

$$\max_{\{\tilde{c}_t,\ell_t,\tilde{a}_{t+1}\}} E\left[Y_0^{\eta(1-\mu)} \sum_{t=0}^{\infty} \left[\beta(1+g)^{\eta(1-\mu)}\right]^t \frac{(\tilde{c}_t^{\eta}\ell_t^{1-\eta})^{(1-\mu)}}{(1-\mu)} |\tilde{a}_0, e_0\right] \tag{15}$$

$$\text{subject to} \quad \tilde{c}_t + (1+g)\tilde{a}_{t+1} \leq (1+\bar{r})\tilde{a}_t + \overline{w}e_t(1-\ell_t) + \chi \tag{16}$$

$$\tilde{c}_t \geq 0, 0 \leq \ell_t \leq 1, \tilde{a}_t \geq 0, t \geq 0 \tag{17}$$

The government budget constraint becomes:

$$\gamma + \chi + rb = (1+g)b - b + \tau_y \left[\frac{w_t N + rK_t}{Y_t} + r(b + prmtl + land + td)\right] \tag{18}$$

and since $\widetilde{w}N + rk = 1 - \delta k$:

$$\gamma + \chi + rb = (1+g)b - b + \tau_y[1 - \delta k + r(b + prmtl + land + td)] \tag{19}$$

The condition of asset market equilibrium is as follows:

$$\bar{a} = k + b + prmtl + land + td \tag{20}$$

Equations (5) and (6) are rewritten as:

$$r = \frac{\theta}{k} - \delta \tag{21}$$

$$\widetilde{w} = \frac{(1-\theta)}{N} \tag{22}$$

where k and $\widetilde{w}$ denote their ratios to output, i.e., $k = \frac{K_t}{Y_t}$, and $\widetilde{w} = \frac{w_t}{Y_t}$.

Using Equation (7), $\frac{K_t}{z_t N}$, and then $k = \frac{K_t}{Y_t}$ can be expressed as a function of r. Further, by using Equation (8), $\frac{w_t N}{Y_t}$ can also be represented as a function of r by:

$$k = \frac{K_t}{Y_t} = \kappa(r) \tag{23}$$

$$\frac{w_t N}{Y_t} = \omega(r) \tag{24}$$

Now, solving the optimization problem and calculating the first-order conditions are discussed. However, there are inequality constraints in the model, making the optimization problem much more difficult. The penalty functions transform constrained problems into unconstrained ones by introducing a penalty for constraint violations. Therefore, the inequality constraints are incorporated into the objective function as follows:

$$E\left[\sum_{t=0}^{\infty}\left(\beta(1+g)^{\eta(1-\mu)}\right)^{t}\left\{\frac{(\tilde{c}_t^{\eta}\ell_t^{1-\eta})^{(1-\mu)}}{(1-\mu)}+\frac{1}{3}\zeta(\min(\tilde{a}_t,0)^3 + \min(1-\ell_t,0)^3)\right\}|\tilde{a}_0,e_0\right] \tag{25}$$

where ζ is positive, and if $\tilde{a}_t < 0$ and $\ell_t > 1$, $-\zeta\tilde{a}_t^3$ and $-\zeta(1-\ell_t)^3$ are subtracted from the consumer's value function, respectively, and have adverse effects.

The Bellman equation reads as:

$$V(\tilde{a},e) = \max_{\{\tilde{c},\ell,\tilde{a}'\}} Y_0^{\eta(1-\mu)}\left\{\frac{(\tilde{c}^{\eta}\ell^{1-\eta})^{(1-\mu)}}{(1-\mu)}+\frac{1}{3}\zeta[\min(\tilde{a},0)^3 + \min(1-\ell,0)^3] + \beta(1+g)^{\eta(1-\mu)}E[V(\tilde{a}',e')|\tilde{a},e]\right\} \tag{26}$$

Therefore, the decision rules $c(x,i)$, $x' = \alpha(x,i)$, and $\ell(x,i)$ that satisfy the following first-order conditions (FOCs) can be calculated by:

$$\eta(1+g)c(x,i)^{\eta(1-\mu)-1}\ell(x,i)^{(1-\eta)(1-\mu)} = \beta(1+g)^{\eta(1-\mu)}\{\textstyle\sum_j \pi_{i,j}\,\eta(1+\bar{r}).\,c(x',j)^{\eta(1-\mu)-1}\ell(x',j)^{(1-\eta)(1-\mu)} + \zeta(\min(\alpha(x,i),0))^2\} \tag{27}$$

$$(1-\eta)c(x,i)^{\eta(1-\mu)}\ell(x,i)^{(1-\eta)(1-\mu)-1} = \eta\bar{w}e(i)c(x,i)^{\eta(1-\mu)-1}\ell(x,i)^{(1-\eta)(1-\mu)} + \zeta\min(1-\ell(x,i),0)^2 \tag{28}$$

$$c(x,i) + (1+g)\alpha(x,i) = (1+\bar{r})x + \bar{w}e(i)\big(1-\ell(x,i)\big) + \chi \tag{29}$$

for $i = 1,\dots,m$ and $x \in [0, x_{max}]$, where $e(i)$ is the productivity level in state i. If $c(x,i)$ in (29) is substituted into (27), and the solution for $\ell(x,i)$ is calculated by Newton's method [that is, $\ell^*(x,i;\alpha)$], then the FOCs (27)-(29) can be written in terms of the residual as follows:

$$R(x,i;\,\alpha) = \eta(1+g)c(\ell^*(x,i;\alpha))^{\eta(1-\mu)-1}\ell^*(x,i;\alpha)^{(1-\eta)(1-\mu)} - \beta(1+g)^{\eta(1-\mu)}\left\{\textstyle\sum_j \pi_{i,j}\,\eta(1+\bar{r}).\,c\big(\ell^*(\alpha(x,i),j;\alpha)\big)^{\eta(1-\mu)-1}.\,\ell^*(\alpha(x,i),j;\alpha)^{(1-\eta)(1-\mu)} + \zeta\min(\alpha(x,i),0)^2\right\} \tag{30}$$

and by rewriting it:

$$R(x,i;\,\alpha) = \eta(1+g)\left\{\frac{1-\eta}{\eta\bar{w}e(i)}\right\}^{(1-\eta)(1-\mu)}\{(\bar{w}e(i) + (1+\bar{r})x - (1+g)\alpha(x,i) + \chi)\eta\}^{-\mu} - \beta(1+g)^{\eta(1-\mu)}\left\{\textstyle\sum_j \pi_{i,j}\,\eta(1+\right. \tag{31}$$

$\bar{r})\left\{\frac{1-\eta}{\eta\bar{w}e(j)}\right\}^{(1-\eta)(1-\mu)}\{(\bar{w}e(j)+(1+\bar{r})\alpha(x,i)-(1+g)\alpha(\alpha(x,j),j)+$

$\chi)\eta\}^{-\mu}+\zeta(\min(\alpha(x,i),0))^2\Big\}$

Then, $\alpha(x,i)$ should be approximated ($\alpha^h(x,i)$) in such a way that $R(x,i;\ \alpha^h)$ approaches zero for all x,i. Weighted residual methods[1] get the residual close to zero in the weighted integral case. This task is done by the FEM[2] as a piecewise application of the weighted residual method (McGrattan, 1999)[3]. It is assumed that $\alpha^h(x,i)$ is defined as a weighted sum of linear basic functions on the element $[x_e,x_{e+1}]$:

$$\alpha^h(x,i)=\psi_e^iN_e(x)+\psi_{e+1}^iN_{e+1}(x) \tag{32}$$

$$N_e(x)=\frac{x_{e+1}-x}{x_{e+1}-x_e},\qquad N_{e+1}(x)=\frac{x-x_e}{x_{e+1}-x_e} \tag{33}$$

By setting the following weighted residual to zero, the problem is solved, and the coefficients ψ_e^i are found:

$$\int R(x,i;\ \alpha^h).N_e(x).dx=0,\qquad i=1,\dots,m,\ e=1,\dots,n \tag{34}$$

Here, the Galerkin method is used, where the set of weight functions is the same as the basic functions.

Invariant Distribution of Assets

To calculate equilibrium prices (r and w), it is necessary to construct a stationary distribution. The solution to the household's optimum savings problem is a policy function $x'=\alpha(x,i)$, that induces a stationary distribution across households of $H(x,i)$[4] (Ljungqvist and Sargent, 2018: 792). Therefore, this distribution can be derived from the decision rule for asset holdings.[5]

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[1]. In this method, the approximate solution to the functional equation is defined as a linear combination of certain basic functions. The objective is to compute the basic function coefficients to approximate the solution. A weighted integral of residual is set to zero to find the coefficients (McGrattan, 1999).

[2]. The finite element method is a piecewise application of the weighted residual method. In the method, first, the domain must be subdivided into non-intersecting subdomains called *elements*. Because the method relies on fitting low-order polynomials on subdomains of the state space instead of high-order polynomials on the entire state space and the result is a sparse system of equations. Finally, the local approximations are gotten together to have a global approximation (McGrattan, 1996; 1999).

[3]. An excellent reference for these methods (i.e., projection methods) is McGrattan (1996; 1999).

[4]. $x=\alpha^{-1}(x',i)$ is constructed from $x'=\alpha(x,i)$.

[5]. In general, it is assumed that an individual's state consists of his current assets' holdings, $a\in[\underline{a},\overline{a}]$, and his current productivity level, $\lambda\in\Lambda=\{\lambda_j,j=1,\dots,n\}$. Hence a member of the state is $(a,\lambda)\in S=[\underline{a},\overline{a}]\times\Lambda$. In addition, suppose that the next period's asset holdings are given by $a'=\alpha'(a,\lambda)$, while the next period's productivity level is governed by a first-order Markov chain with probability transition matrix *P*. The element of *P* that gives the probability of getting λ' next period

To calculate the stationary distribution, $H(x,i) = Pr(x_t < x|e_t = e(i))$, the following functional equation is solved:

$$H_{n+1}(x',j) = \sum_{i=1}^{m} \pi_{i,j} \,. H_n(\alpha^{-1}(x',i),i).I(x' \geq \alpha(0,i)) \tag{35}$$

and if it is assumed $H_n = H_{n+1}$, the functional equation for the stationary distribution is:

$$H(x',j) = \sum_{i=1}^{m} \pi_{i,j} \,. H(\alpha^{-1}(x',i),i).I(x' \geq \alpha(0,i)) \tag{36}$$

where I is an indicator function to apply borrowing constraint, so if $x' \geq \alpha(0,i)$, it is equal to one and otherwise is equal to zero.

Equilibrium

A stationary equilibrium is a policy function $\alpha(x,i)$, a stationary distribution $H(x,i)$, aggregate hours *N*, prices *r*, *w*, and mean asset holdings $E(\tilde{a}_t)$. So far, it is assumed that $\alpha(x,i)$ and $H(x,i)$ have been calculated. Then, an algorithm for calculating the interest rate and total hours is presented. First, a guess is considered for *N*, which is derived from the following model:

$$N = \frac{1}{1+\frac{(1-\eta)}{[\eta(1-\tau_y)(1-\theta)]}.(1-\gamma-\theta\frac{g+\delta}{r+\delta})} \tag{37}$$

Then, an interval for the equilibrium interest rate as $[r_l, r_u]$ is considered (according to the real rate of return of the economy), and the initial guess for the interest rate is $\frac{r_l+r_u}{2}$.

Also, the proportional tax rate is derived from the government budget constraint as follows:

$$\tau_y = \frac{\gamma+\chi+(r-g)b}{1-\delta\frac{\theta}{r+\delta}+r(b+prmtl+land+td)} \tag{38}$$

The after-tax interest rate and the after-tax wage rate are calculated as:

$$\bar{r} = (1-\tau_y)r \tag{39}$$

given λ today is denoted by $\pi(\lambda'|\lambda)$ or $p_{i,j} = Prob(\lambda' = \lambda_j|\lambda = \lambda_i)$. The stationary distribution is $F(a,\lambda)$, that satisfies:

$$F(a',\lambda') = \sum_{\lambda\in\Lambda} \pi(\lambda'|\lambda)F(a'^{-1}(a',\lambda),\lambda)$$

for all $(a'\lambda') \in S$. To compute the stationary equilibrium, it can either compute the distribution function, $F(a,\lambda)$, or the density function, $f(a,\lambda)$. Note that for these calculations, it is necessary to have an approximation of the decision rule for savings, $\alpha'(a,\lambda)$ (Kopecky, 2007).

$$\bar{w} = (1-\tau_y)\frac{\omega(r)}{N} \tag{40}$$

Now, we can calculate the mean asset holdings $E(\tilde{a}_t)$ as follows:

$$E(\tilde{a}_t) = \frac{1}{2}\sum_j \sum_i (\widehat{H}(i+1,j) - \widehat{H}(i,j))(x(i+1) + x(i)) \tag{41}$$

In these models, for a given interest rate *r*, there are two interpretations for the mean. *First*, it is the average asset held by an agent (across time). *Second*, it is the average asset held by the total economy [average across agents indexed by (x, e)]. The basis of what is called the 'Bewley models' is to make *r* an equilibrium object that adjusts to set $E(\tilde{a}_t)$ equal to a particular value (Ljungqvist and Sargent, 2018: 790). The value to which $E(\tilde{a}_t)$ must be equated through an appropriate adjustment of *r* is:

$$\bar{a} = \frac{\theta}{r+\delta} + b + prmtl + land + td \tag{42}$$

Therefore, it should be checked until the markets clear; i.e.,

$$E(\tilde{a}_t) = \frac{\theta}{r+\delta} + b + prmtl + land + td \tag{43}$$

Thus, if the right side of Equation (43) is more than the left side, then $r_l = r$ is set, and otherwise, $r_u = r$ is set. Iterations will continue until Equation (43) is satisfied. Then, the estimate for *N* will be updated.

Finally, the welfare criterion has been used to investigate the effect of increment in the debt level on welfare:

$$\Omega = Y_0^{\eta(1-\mu)}.\iint V(a,e).\,dH(a,e) \tag{44}$$

where, $V(a, e)$ is the optimum value function, and $H(a, e)$ is the invariant joint distribution of assets and productivity. Indeed, the welfare gain at the current debt ratio is set equal to zero, and the welfare gain or loss of being in the other debt ratios is measured compared to the current ratio in terms of consumption.

Parameterization of the Benchmark Model

In this section, the model was calibrated to the Iranian economy for the 1992–2020 period[1]. In the model, one time period is considered one year. The parameter values are presented in Table 2. In calibration, the parameter values can be obtained from previous research or estimated from data (Kydland and Prescott, 1996). The

[1]. Most of the computation is based on runs of Fortran code with Visual Studio 2012 and Intel Fortran Compiler version 2015.

estimation and calculation procedures, and the source of each parameter are described in Table 2's footnote. Since the estimation of productivity parameters requires further explanation, the calculation method is elaborated as follows.

One important category of parameters pertains to the idiosyncratic productivity shocks of each agent. The stochastic process for e_t is specified as $\ln(e_t)$, which is assumed to be an AR(1) process, with a serial correlation coefficient ρ and a standard deviation σ. These parameters for idiosyncratic productivity are estimated using panel data on individual ages. However, in developing countries including Iran, the Household Expenditure and Income Survey (HEIS) are collected cross-sectionally, and it is not possible to access panel surveys. Therefore, it is necessary to use pseudo-panel methods for studying individual income dynamics. According to Deaton (2019: 116–117), the survey data of the Statistical Centre of Iran has been used to follow the individual's cohorts over time for calculating the average earnings, where cohorts are defined by the date of birth.[1] These averages relate to the same group of people, and subsequently have many properties of panel data. Then, the annual average of individual labor incomes[2] of seven cohorts was extracted for the time series of 2001 to 2020. These nominal values were weighted by the consumer price index (CPI) to get real income. Finally, the individual income process is estimated, and the average of cross-sectional coefficient estimates is calculated (ρ and σ_ε). Then, Tauchen's (1986) method is used to approximate the first-order auto-regression of $\ln(e_t)$ with a Markov chain with seven states. Thus, there are the values of the seven states and a matrix of probability transition denoted by π as the Markov process parameters. The parameters are obtained by running the codes in MATLAB.

1. For instance, for the cohort born in 1976, who turned 25 in 2001, we used the 2001 survey data to calculate the average earnings of 25-year-olds, and the result is recorded as the first point. Subsequently, we calculated the average earnings of 26-year-olds from the 2002 survey and the second point is obtained. The rest of the points comes from the other surveys, monitoring the cohort born in 1981 through the 20 surveys until they are last observed at age 44 in 2020 and then first cohort is completed. The same process is followed for seven cohorts, born in 1976, 1971, 1966, 1961, 1956, 1951, and 1946.

2. The labor incomes are calculated by summing up the following items, which are available in the Household Expenditure and Income Survey of Iran: gross monetary income from wages and salaries of government and private sectors, monetary income from non-agricultural freelance occupations, miscellaneous monetary incomes (pensions and unemployment benefits), gross monetary income from wages and salaries of the cooperative sector.

Table 2. Benchmark Model Parameter Values and Sources

Parameter	Symbol	Source	Value
Capital's share of output	θ	Esmailipour Masouleh, Shirinbakhsh and Ebrahimi (2018)	0.39000
Discount factor	β	Research finding[1]	0.99566
Capital depreciation rate	δ	Ostadzad and Behpour (2014)	0.05100
Preferences parameter (The share of consumption)	η	Research finding[2]	0.26432
Coefficient of relative risk aversion	μ	Komijani and Tavakkolian (2012)	1.50000
Government consumptions-to-GDP ratio	γ	Research finding[3]	0.25767
Growth of per capita GDP	g	Research finding[4]	0.01199
Debt-to-GDP ratio	b	International Monetary Fund (IMF)[5]	0.39530
Transfer payments-to-GDP ratio	χ	Research finding[6]	0.01923
Total hours for the major groups of economic activity divided by the 10+population (2013–2018) and 15+population (after 2019)	N	Research finding[7]	0.26059
Persistence of idiosyncratic productivity	ρ	Research finding	0.42000
The volatility of idiosyncratic productivity shocks	σ_ε	Research finding	0.10000
Real return	r	Research finding [8]	0.02635
Precious metals to GDP	$prmtl$	Iranian Central Bank's Economic Accounts Office[9]	1.19600
Land to GDP	$land$	Iranian Central Bank's Economic Accounts Office[10]	3.39497
Term deposits to GDP	td	Central Bank of Iran[11]	0.64926
Proportional income tax	τ_y	Research finding[12]	0.31702

1. $\beta = \frac{1}{(1+g)^{\eta(1-\mu)-1}(1+\bar{r})}$, (**Source:** derived from the model);

2. $\eta = \left[1 + (1-\tau_y)\frac{(1-N)(1-\theta)}{N\left[1-\gamma-\theta\frac{(g+\delta)}{(r+\delta)}\right]}\right]^{-1}$, (**Source:** derived from the model);

3. Government consumptions = Public consumption expenditures + Public sector gross fixed capital formation, 1992–2020 (**Source**: National Account of Iran);

4. The average growth rate of per capita GDP (constant 2015 prices), 1992–2020 (**Source**: National Account of Iran);

5. The ratio of central government debt to GDP, 2020;

6. Transfer payments = Sum of subsidies and grants, 2000–2020 [**Source**: Government Financial Statistics (GFS) (2000–2016), and Annual Budget Law (2017–2020)];

7. **Source**: Statistical Centre of Iran (SCI), Labor Force Census Plan, 2005–2020;

8. The average growth rate of GDP (constant 2016 prices), 1992–2020 (**Source**: National Account of Iran);

9. Precious metals (gold and gold coins) to GDP ratio, 2020 (**Source**: National Balance Sheet of Iran);

10. Land (the value of land under residential and nonresidential units, and agricultural and forest lands) to GDP, 2020 (**Source**: National Balance Sheet of Iran);

11. Short-term and long-term deposits to GDP ratio, 2020;

12. $\tau_y = \frac{\gamma+\chi+(r-g)b}{1-\delta\frac{\theta}{r+\delta}+r(b+prmtl+td)}$, (**Source:** derived from the model);

In the next section, the findings for the benchmark parameters are described. In Section 6, the robustness of the results to changes in the δ and θ values in the estimated ranges, and calculations in different databases and studies, is discussed. These parameters could potentially influence the results.

Result and Discussion

The main findings of this study are presented in Figure 1. The figure for the "benchmark case" shows the plots of the welfare gain, the interest rates, the proportional tax rate, and total hours versus the debt ratio for the benchmark parameters. According to the welfare gain plot, the optimum quantity of debt is significantly negative, and the welfare gain (measured as a percent of per capita consumption) from moving to the optimum is about 2.66% of per capita consumption, which is a significant amount. Thus, the negative role that debt plays in the model (i.e., crowding out private capital) is more than the positive role of enhancing liquidity. The optimal interest rate (i.e., 2.53%) is lower than the interest rate at the current value of the debt/GDP ratio (i.e., 2.63%). The optimal tax rate (i.e., 28.89%) is also lower than its current value of 31.7%. The negative optimal debt ratio implies that it is efficient for the private sector to spend, because government spending is consumption spending, whereas private sector spending is in the form of investment, which increases production and total consumption, and ultimately increases welfare. Indeed, the level of public debt prevents investment and growth through a 'crowding out' channel.

The optimum quantity of government debt can be negative in the long run, although it is not observed in the long historical data. In other words, it is optimal that the government accumulates assets in the long run, so that all government expenditures are financed through the interest income of private assets, without resorting to distortionary taxes.

One significant difference in the results of the current study compared to other studies is that the optimal debt ratio in Iran is much lower and more negative. This is because in this study, the idiosyncratic wage risk has been set based on the calculations using the survey data of Iran: $\rho = 0.42$, $\sigma\varepsilon = 0.1$, *indicating lower wage risk compared to other studies such as Aiyagari and McGrattan (1994, 1998)* ($\rho = 0.6$, $\sigma\varepsilon = 0.3$), Flodén (2001) ($\rho = 0.9$, $\sigma\varepsilon = 0.21$), *and Nakajima and Takahashi (2017)* ($\rho = 0.9$, $\sigma\varepsilon = 0.1754$). Indeed, households face lower wage risk and the insurance role of government debt decreases in our study. Therefore, the optimal quantity of debt becomes lower and more negative compared to other studies.

Although the debt-to-GDP ratio of −420% maximizes the welfare, it seems challenging to achieve this ratio in reality, given that the current ratio is around 39.35% in Iran and is still increasing. However, Figure 1 demonstrates that decreasing the debt-to-GDP ratio from 39.35% to zero results in a significant increase of per-capita welfare (i.e., around 0.47%). Therefore, transitioning to the debt-to-GDP ratio of zero would be an immediate and feasible goal to enhance efficiency in present-day Iran.

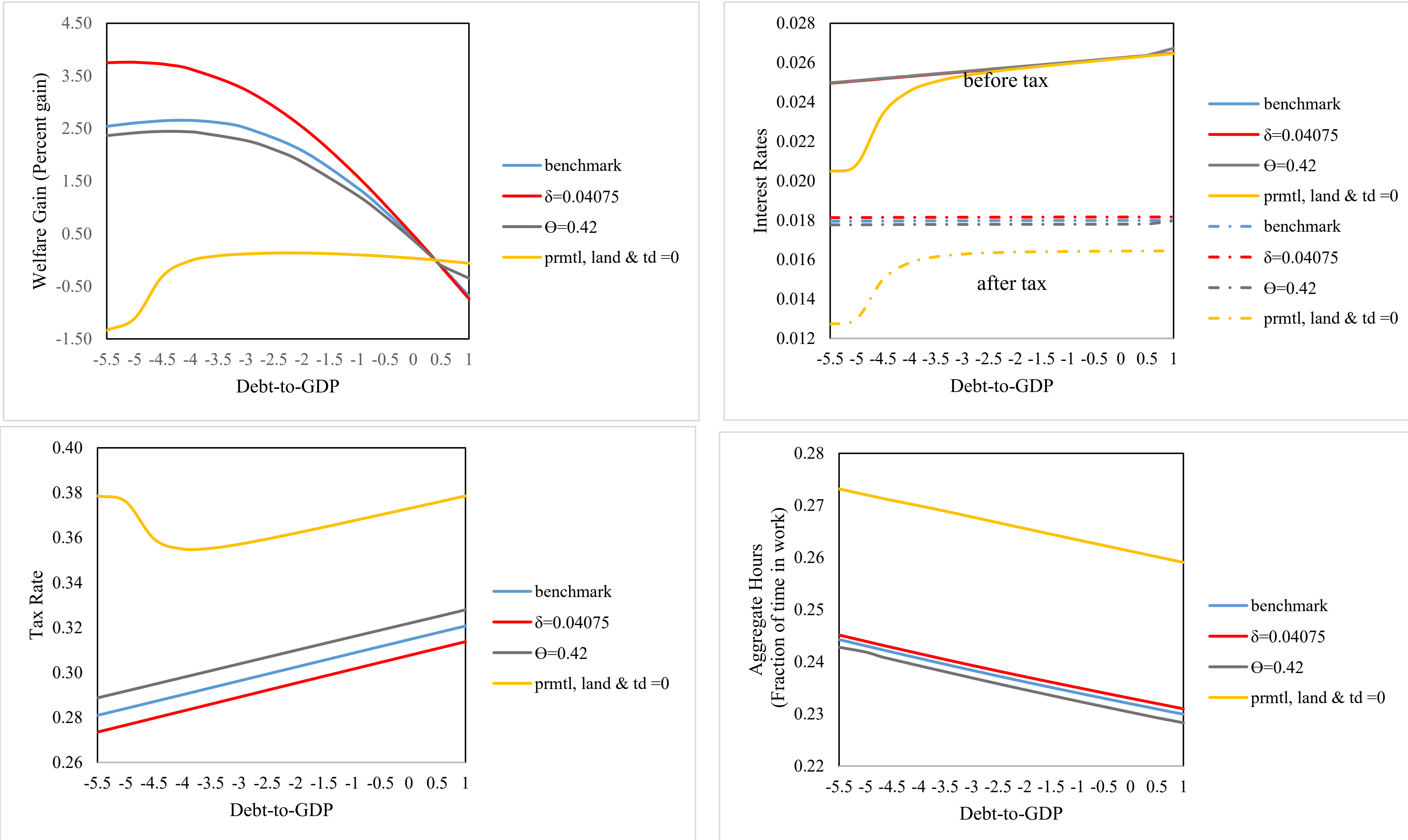

Figure 1. Welfare Gain, Interest Rates, Proportional Tax Rate, and Total Hours Versus the Debt Ratio (Four Scenarios)

Source: Research finding.

Robustness of the Results

This section examines the robustness of the primary analysis results to changes in some parameters: the capital depreciation rate δ and capital's share of output, θ. These parameters change in the range of estimates and calculations in different databases and studies. It is also examined how the results change when precious metals, land, and term deposits are excluded and households hold only government debt and capital as in Aiyagari and McGrattan (1998). The benchmark and new parameter values are listed in Table 3. The parameters could potentially influence the results. As presented in the last rows of Table 3, the results are similar to the results of the benchmark model parameters, and in any case, the negative effect of debt is significant. The results are also presented in Figure 1. Therefore, similar to the studies of Aiyagari and McGrattan (1998) under a calibration with a higher interest rate than the benchmark for the United States and Nakajima and Takahashi (2017) under their benchmark calibration for Japan, our findings showed that having positive government assets is optimal. In comparison with the studies conducted in Iran regarding the calculation of the optimal debt ratio, the current study has utilized a model that incorporates Iran's specific conditions, leading to results indicating a significant gap between the current debt ratio and the optimal ratio. Furthermore, considering the fourth scenario (prmtl, land & td=0), it can be noted that if the excessive asset holding by households is restricted (e.g., through policies such as imposing taxes on holding these assets, developing financial markets that will lead to a reduction in borrowing constraints, etc.), the welfare loss from maintaining the current level of debt instead of the optimal quantity decreases in comparison to other scenarios.

Table 3. Changes in parameter values relative to Benchmark Case

Parameter symbol	Benchmark	$\delta = 0.04075$[2]	θ =0.42[1]	prmtl, land & td=0
θ	0.39000	0.39000	**0.42000**	0.39000
β	0.99566	**0.99551**	**0.99585**	**0.99727**
δ	0.05100	**0.04075**	0.05100	0.05100
η	0.26432	**0.26733**	**0.26467**	**0.28197**
μ	1.50000	1.50000	1.50000	1.50000
γ	0.25767	0.25767	0.25767	0.25767
g	0.01199	0.01199	0.01199	0.01199
b	0.39530	0.39530	0.39530	0.39530
χ	0.01923	0.01923	0.01923	0.01923
N	0.26059	0.26059	0.26059	0.26059
ρ	0.42000	0.42000	0.42000	0.42000
σ_ε	0.10000	0.10000	0.10000	0.10000
R	0.02635	0.02635	0.02635	0.02635
$prmtl$	1.19600	1.19600	1.19600	**0.00000**
$land$	3.39497	3.39497	3.39497	**0.00000**
td	0.64926	0.64926	0.64926	**0.00000**
τ_y	0.31702	**0.34369**	**0.32421**	**0.37513**
Optimal debt to GDP ratio	**-4.20**	**-5.10**	**-4.30**	**-2.25**
Welfare Gain in terms of Consumption (%)	**2.65579**	**3.76014**	**2.44603**	**0.13467**

Source: Research finding.

[1]. Komijani and Tavakkolian (2012).
[2]. Using the weighted ratio of fixed capital consumption to capital stock (of machinery and buildings) (constant 2015 prices), 2011–2020, (**Source:** Research finding).

Conclusion and Recommendation

The purpose of the present study has been to investigate the government debt ratio and its impacts on welfare for Iran's economy. Since households in Iran cannot easily access bank loans in times of necessity, they consistently save a significant portion of their income as precautionary savings. This excessive saving is imposed on households' budgets due to borrowing constraints. Therefore, the appropriate model for calculating the optimal debt level is an Aiyagari (1994)-style model, which has been extended according to the economic and social conditions of Iran. Indeed, given that Iran's economy faces high inflation in addition to borrowing constraints and due to the absence of taxes on asset holdings, Iranian households prefer to hold assets such as gold, land, and term deposits, in addition to government debt and physical capital, for precautionary savings purposes.

According to the welfare criteria, the optimum quantity of debt is significantly negative, and the welfare gain (measured as a percent of per capita consumption) from moving to the optimum is about 2.66% of per capita consumption. Furthermore, the results of assessing the resilience of the primary findings with respect to specific parameters resemble those of the benchmark parameterization. These findings indicate that the negative effect of debt in the model (i.e., crowding out private capital) is more than the positive effect (i.e., enhancing liquidity). The negative optimal debt ratio implies that it is efficient for the private sector to spend. As a result, increasing private sector spending will improve welfare, while limiting government spending and maintaining positive government assets are optimal. It should be noted that although a high negative debt-to-GDP ratio maximizes the welfare, it seems challenging to achieve this ratio in reality, considering that the current ratio is around 39.35% in Iran and is still increasing. However, the results show that a decrease in the debt-to-GDP ratio from the current level to zero results in a significant increase of per-capita welfare (around 0.47% of per capita consumption). Hence, transitioning to the debt-to-GDP ratio of zero would be an immediate and feasible goal to enhance efficiency in present-day Iran. Therefore, it is optimal that the government accumulate assets in the long run, so that all government expenditures are financed through the interest income of private assets, without resorting to distortionary taxes. Furthermore, if through policies such as imposing taxes on holding the risk-free assets, and by developing financial markets to reduce credit constraints, the excessive asset holdings by households are restricted, the welfare loss from maintaining the current level of debt instead of the optimal quantity will decrease.

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